III. Calcular, utilizando integrales, el área de la región entre las curvas: (40 puntos)

$y = x + 2$

$y = -x^2 + 4$

$x = -3$

Ptos 1.

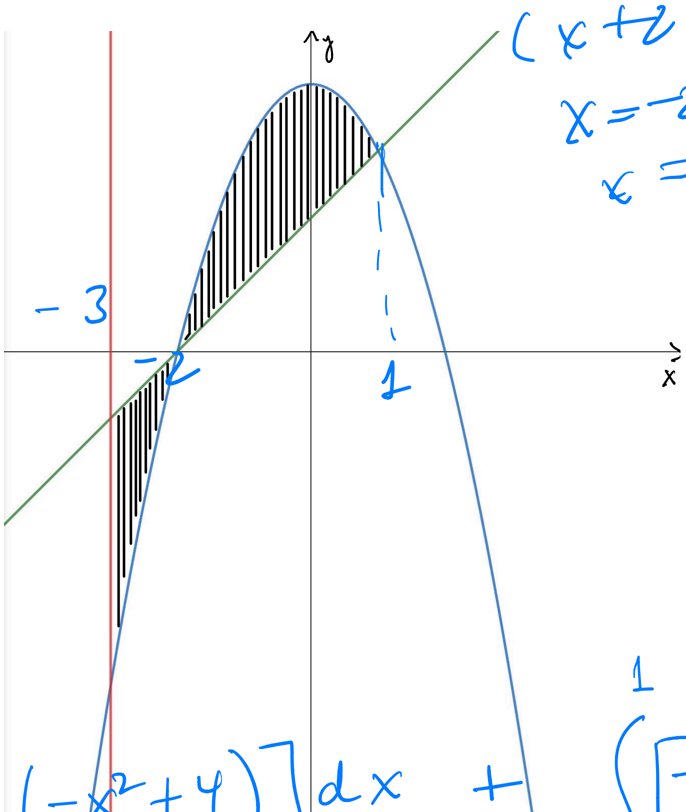
$x + 2 = -x^2 + 4$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2$

$x = 1$



$$A = \int_{-3}^{-2} \left[x+2 - (-x^2+4) \right] dx + \int_{-2}^{1} \left[-x^2+4-(x+2) \right] dx$$

$$A = \int_{-3}^{-2} (x^2 + x - 2)\, dx + \int_{-2}^{1} (-x^2 - x + 2)\, dx$$

$$A = \left(\frac{x^3}{3} + \frac{x^2}{2} - 2x \right) \Big|_{-3}^{-2} + \left(-\frac{x^3}{3} - \frac{x^2}{2} + 2x \right) \Big|_{-2}^{1}$$

$$A = \left[\left(-\frac{8}{3} + 2 + 4 \right) - \left(-9 + \frac{9}{2} + 6 \right) \right] + \left[\left(-\frac{1}{3} - \frac{1}{2} + 2 \right) - \left(\frac{8}{3} - 2 - 4 \right) \right]$$

$$\frac{11}{6} + \frac{9}{2} = \frac{19}{3}\, u^2$$

$$\int_{-2}^{2} f(x)\,dx =$$

Donde: $f(x) = \begin{cases} x^2 & ; -2 < x < 0 \\ |x-1| & ; 0 \leq x < 2 \end{cases}$

$$= \int_{-2}^{0} x^2\,dx + \int_{0}^{1} -(x-1)\,dx + \int_{1}^{2} (x-1)\,dx$$

$$= \left(\frac{x^3}{3}\right)\Big|_{-2}^{0} + \left(-\frac{x^2}{2} + x\right)\Big|_{0}^{1} + \left(\frac{x^2}{2} - x\right)\Big|_{1}^{2}$$

$$= \left[0 - \left(-\frac{8}{3}\right)\right] + \left[\left(-\frac{1}{2} + 1\right) - 0\right] + \left[\left(2 - 2\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \frac{8}{3} + \frac{1}{2} + \frac{1}{2}$$

$$= \frac{11}{3} \,//$$

2. $\int_{1}^{4} \frac{1}{(x-1)^{1/3}} \mathrm{d}x =$

$x \neq 1$

- $\lim_{a \to 1^+} \int_{a}^{4} \frac{1}{(x-1)^{1/3}} dx$

- $\int \frac{1}{(x-1)^{1/3}} dx =$ $\quad u = x-1$, $du = dx$

$\int u^{-1/3} dx = \frac{3}{2} u^{2/3} = \frac{3}{2}(x-1)^{2/3}$

- $\lim_{a \to 1^+} \left(\frac{3}{2}(x-1)^{2/3} \right) \Big|_{a}^{4}$

- $\lim_{a \to 1^+} \left(\frac{3}{2} \cdot 4^{2/3} - \frac{3}{2}\underbrace{(a-1)^{2/3}}_{0} \right)$

$= \frac{3}{2} \cdot 3^{2/3}$

$= \frac{3}{2}\sqrt[3]{9}$ $\therefore$ la integral converge

1. $\int_{-\infty}^{1} e^{2x} dx =$

- $\lim_{a \to -\infty} \int_a^1 e^{2x} dx$
- $\int e^{2x} dx = \frac{e^{2x}}{2}$
- $\lim_{a \to \infty} \left(\frac{e^{2x}}{2} \right) \Big|_a^1$
- $\lim_{a \to \infty} \left(\frac{e^2}{2} - \cancel{\frac{e^{2a}}{2}} \right)$ 0

$= \frac{e^2}{2}$ ∴ la integral converge