Encontrar $\int \frac{\ln(x)}{x}\,dx$

$$\int \ln(x)\cdot\frac{1}{x}\,dx$$

ILATE

$x>0$

$\mathrm{Dom}(f)=\mathbb{R}^+$

$u=\ln(x)$

$du=\frac{1}{x}\,dx$

$dv=\frac{1}{x}\,dx$

$v=\ln|x|=\ln(x)$

$$\underbrace{\int \ln(x)\cdot\frac{1}{x}\,dx}=\overset{u}{\ln(x)}\cdot\overset{v}{\ln(x)}-\underbrace{\int \overset{v}{\ln(x)}\cdot\overset{du}{\frac{1}{x}\,dx}}$$

$$2\int \ln(x)\cdot\frac{1}{x}\,dx=(\ln(x))^2$$

$$\int\frac{\ln(x)}{x}\,dx=\frac{(\ln(x))^2}{2}+C\ ,\quad C\in\mathbb{R}$$

X 8. $\int \frac{\ln(\sqrt{x})}{\sqrt{x}}dx$

X 9. $\int x\log(x)dx$

10. $\int xe^x dx$

11. $\int xe^{mx}dx$

12. $\int (2x+1)e^{3x}dx$

13. $\int \ln\left(x^x\right)dx$

14. $\int x^2e^x dx$

15. $\int x^3e^{x^2}dx$

10

ILATE

$u = x \qquad dv = e^x dx$

$du = 1\,dx \qquad v = e^x$

$$\int x\cdot e^x dx = \underset{u}{x}\cdot \underset{v}{e^x} - \int \underset{v}{e^x}\, \underset{du}{dx}$$

$$= xe^x - e^x + C$$

$$= e^x(x-1) + C, \qquad C\in\mathbb{R}$$

$$\boxed{\begin{array}{c}\text{ILATE} \\ \uparrow\ \ \uparrow \\ x\ \ e^x\end{array}}$$

$u \quad dv$

8. $\int \frac{\ln(\sqrt{x})}{\sqrt{x}}dx$

9. $\int x\log(x)dx$

10. $\int xe^x dx$

11. $\int xe^{mx}dx$

12. $\int (2x+1)e^{3x}dx$

13. $\int \ln(x^x)\,dx$

14. $\int x^2e^x dx$

15. $\int x^3e^{x^2}dx$

11

$$u = x \qquad dv = e^{mx}dx$$

$$du = 1\,dx \qquad v = \frac{e^{mx}}{m}$$

$$\int x\cdot e^{mx}dx = \overset{u}{x}\cdot \overset{v}{\frac{e^{mx}}{m}} - \int \overset{v}{\frac{e^{mx}}{m}}\,\overset{du}{dx}$$

$$= x\frac{e^{mx}}{m} - \frac{1}{m}\int e^{mx}dx$$

$$= x\cdot\frac{e^{mx}}{m} - \frac{1}{m}\cdot\frac{e^{mx}}{m} + c, \quad c\in\mathbb{R}$$

$$= e^{mx}\left(\frac{x}{m} - \frac{1}{m^2}\right) + c, \quad c\in\mathbb{R}$$

8. $\int \frac{\ln(\sqrt{x})}{\sqrt{x}} dx$

9. $\int x \log(x) dx$

10. $\int xe^x ax$

11. $\int xe^{mx} dx$

12. $\int (2x+1)e^{3x} dx$

13. $\int \ln(x^x)\, dx$

14. $\int x^2 e^x dx$

15. $\int x^3 e^{x^2} dx$

12

$u = 2x+1 \qquad dv = e^{3x} dx$

$du = 2dx \qquad v = \frac{e^{3x}}{3}$

$$\int (2x+1)\cdot e^{3x} dx = \underbrace{(2x+1)}_{u}\cdot \underbrace{\frac{e^{3x}}{3}}_{v} - \int \underbrace{\frac{e^{3x}}{3}}_{v} \cdot \underbrace{2dx}_{du}$$

$$= (2x+1)\frac{e^{3x}}{3} - \frac{2}{3}\int e^{3x} dx$$

$$*** \quad = (2x+1)\frac{e^{3x}}{3} - \frac{2}{3}\cdot\frac{e^{3x}}{3} + C \quad ***$$

$$= e^{3x}\left(\frac{(2x+1)}{3} - \frac{2}{9}\right) + C$$

$$= e^{3x}\left(\frac{2x}{3} + \frac{1}{9}\right) + C \quad , C \in \mathbb{R}$$

13 $\int x \cdot \ln(x) dx$

ILATE

8. $\int \frac{\ln(\sqrt{x})}{\sqrt{x}} dx$

9. $\int x \log(x) dx$

10. $\int x e^{x} ax$

11. $\int x e^{mx} dx$

12. $\int (2x+1)e^{3x} dx$

13. $\int \ln(x^{x})\, dx$

14. $\int x^2 e^{x} dx$

15. $\int x^3 e^{x^2} dx$

14

$u = x^2$ | $dv = e^x\, dx$ | INTEGRAL POR PARTES #1

$du = 2x\,dx$ | $v = e^x$

INTEGRAL POR PARTES #2

$u = x$	$dv = e^x dx$
$du = dx$	$v = e^x$

$$\int x^2 e^x dx = x^2 \cdot e^x - \int e^x \cdot 2x\, dx$$

$$= x^2 e^x - 2 \underbrace{\int x \cdot e^x dx}_{u \cdot v - \int v \cdot du}$$

$$= x^2 e^x - 2\cdot\left(x \cdot e^x - \int e^x dx\right)$$

$$= x^2 \cdot e^x - 2(x e^x - e^x) \quad + c \,, \; c \in \mathbb{R}$$

$$= x^2 e^x - 2x e^x + 2 e^x \quad + c \,, \; c \in \mathbb{R}$$

8. $\int \frac{\ln(\sqrt{x})}{\sqrt{x}} dx$

9. $\int x \log(x) dx$

10. $\int x e^x a x$

11. $\int x e^{mx} dx$

12. $\int (2x+1) e^{3x} dx$

13. $\int \ln(x^x)\, dx$

14. $\int x^2 e^x dx$

15. $\int x^3 e^{x^2} dx$

$\boxed{15}$

$$\int \underbrace{x^2}_{u} \cdot \underbrace{x e^{x^2} dx}_{dv}$$

$u = x^2$ | $dv = x e^{x^2} dx$ *

$du = 2x\,dx$ | $v = \frac{1}{2} e^{x^2}$

CAMBIO DE VARIABLE

$$\int x \cdot e^{x^2} dx = \int e^{x^2} \cdot x\,dx = \int e^t \cdot \frac{dt}{2} = \frac{1}{2} \cdot e^t = \frac{1}{2} \cdot e^{x^2}$$

$t = x^2$

$dt = 2x\,dx$

$\frac{dt}{2} = x\,dx$

$$= u\, v - \int v\, du$$

$$= x^2 \cdot \frac{1}{2} e^{x^2} - \int \frac{1}{2} e^{x^2} \cdot 2x\,dx$$

$$= \frac{1}{2} x^2 e^{x^2} - \int e^{x^2} \cdot x\,dx$$

$$= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C = \frac{e^{x^2}}{2}\left(x^2 - 1\right) + C, \quad C \in \mathbb{R}$$

17. $\int xe^{-x}dx$

18. $\int y^3\ln(y)dy$

19. $\int \ln(4x)dx$

20. $\int 15x\sqrt{x+1}dx$

21. $\int \frac{x}{(2x+1)^2}dx$

22. $\int \frac{\ln x}{x^2}dx$

23. $\int_1^2 4xe^{2x}dx$

24. $\int_0^1 xe^{-x^2}dx$

25. $\int_1^2 \frac{3x}{\sqrt{4-x}}dx$

17

$u = x \qquad dv = e^{-x}dx$

$du = dx \qquad v = \frac{e^{-x}}{-1} = -e^{-x}$

$\int e^{\alpha x}dx = \frac{e^{\alpha x}}{\alpha}$

$$\int x\cdot e^{-x}dx = \underset{u}{x}\cdot \underset{v}{-e^{-x}} - \int \underset{v}{-e^{-x}}\,\underset{du}{dx}$$

$$= -x\cdot e^{-x} + \int e^{-x}dx$$

$$= -x\cdot e^{-x} - e^{-x} + c, \quad c\in\mathbb{R}$$

$$= -\frac{x+1}{e^x} + c, \quad c\in\mathbb{R}$$

ILATE

17. $\int xe^{-x}dx$

18. $\int y^3 \ln(y)dy$

19. $\int \ln(4x)dx$

20. $\int 15x\sqrt{x+1}dx$

21. $\int \frac{x}{(2x+1)^2}dx$

22. $\int \frac{\ln x}{x^2}dx$

23. $\int_1^2 4xe^{2x}dx$

24. $\int_0^1 xe^{-x^2}dx$

25. $\int_1^2 \frac{3x}{\sqrt{4-x}}dx$

18

$u = \ln(y)$ | $dv = y^3 dy$

$du = \frac{1}{y} dy$ | $v = \frac{y^4}{4}$

$$\int y^3 \ln(y)\,dy = \ln(y)\cdot\frac{y^4}{4} - \int \frac{y^4}{4}\cdot\frac{1}{y}\,dy$$

$$= \ln(y)\frac{y^4}{4} - \frac{1}{4}\int y^3\,dy$$

$$= \ln(y)\frac{y^4}{4} - \frac{1}{16}y^4 + c \quad , c \in \mathbb{R}$$

17. $\int xe^{-x}dx$

18. $\int y^3 \ln(y)dy$

19. $\int \ln(4x)dx$

20. $\int 15x\sqrt{x+1}dx$

21. $\int \frac{x}{(2x+1)^2}dx$

22. $\int \frac{\ln x}{x^2}dx$

23. $\int_1^2 4xe^{2x}dx$

24. $\int_0^1 xe^{-x^2}dx$

25. $\int_1^2 \frac{3x}{\sqrt{4-x}}dx$

19

$u = \ln(4x)$ | $dv = 1\,dx$

$du = \frac{1}{4x} \cdot 4 = \frac{1}{x}\,dx$ | $v = x$

$$\int \ln(4x)\cdot 1\,dx = \underbrace{\ln(4x)}_{u}\cdot \underbrace{x}_{v} - \int \underbrace{x}_{v}\cdot \underbrace{\frac{1}{x}\,dx}_{du}$$

$$= \ln(4x)\cdot x - \int 1\,dx$$

$$= \ln(4x)\cdot x - x + c, \quad c \in \mathbb{R}$$