

Resolución Prueba 5 matemáticas

ita
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2º ELO
272

Pregunta 1

$$y = \sin(\ln x) + \cos(\ln x)$$

$$y' = \frac{\cos(\ln x)}{x} - \frac{\sin(\ln x)}{x}$$

$$y'' = \left[\frac{-\cancel{\sin(\ln x)} \cdot x - \cos(\ln x) \cdot 1}{x^2} \right] - \left[\frac{\cancel{\cos(\ln x)} \cdot x - \cancel{\sin(\ln x)} \cdot 1}{x^2} \right]$$

$$y'' = -\frac{2\cos(\ln x)}{x^2}$$

calculo de ecuación diferencial

$$\bullet \frac{d^2 y}{dx^2} \cdot x^2 + \frac{dy}{dx} \cdot x + y = 0$$

$$-\frac{2\cos(\ln x)}{x^2} \cdot x^2 + (\cancel{\cos(\ln x)} - \cancel{\sin(\ln x)}) \cdot x + \sin(\ln x) + \cos(\ln x) = 0$$

$$0 = 0$$

R// La función $y = \sin(\ln(x)) + \cos(\ln(x))$ SI satisface la ecuación diferencial de $y'' \cdot x^2 + y' \cdot x + y = 0$.

Pregunta 2

Pregunta $\frac{dy}{dx}$ usando derivacion logaritmica

a) $y = 3^{\tan(x^3+x)} / \ln$

$$\ln(y) = \tan(x^3+x) \cdot \ln(3)$$

$$\frac{y'}{y} = \sec^2(x^3+x) \cdot (3x^2+1) \cdot \ln(3)$$

$$y' = \left[\sec^2(x^3+x) \cdot (3x^2+1) \cdot \ln(3) \right] \cdot 3^{\tan(x^3+x)}$$

b) $y = \frac{(e^x - x)^{10} \cdot (8 + x^3)^2}{\sqrt{6 + \sqrt{x}}} / \ln$

$$\ln(y) = 10 \cdot \ln(e^x - x) + 2 \ln(8 + x^3) - \frac{1}{2} \ln(6 + \sqrt{x})$$

$$\frac{y'}{y} = 10 \cdot \frac{1}{e^x - x} \cdot (e^x - 1) + 2 \cdot \frac{1}{8 + x^3} \cdot (0 + 3x^2) - \frac{1}{2} \cdot \frac{1}{(6 + \sqrt{x})} \cdot \left(0 + \frac{1}{2\sqrt{x}}\right)$$

$$\frac{y'}{y} = \frac{10e^x - 10}{e^x - x} + \frac{6x^2}{8 + x^3} - \frac{1}{(12 + 2\sqrt{x})} \cdot \left(\frac{1}{2\sqrt{x}}\right)$$

$$y' = \left[\frac{10e^x - 10}{e^x - x} + \frac{6x^2}{8 + x^3} - \frac{1}{24\sqrt{x} + 4x} \right] \cdot \left[\frac{(e^x - x)^{10} \cdot (8 + x^3)^2}{\sqrt{6 + \sqrt{x}}} \right]$$

$$c) \quad y = \frac{\sin(x) \cdot (x^2+x)^{3x}}{110} / \ln$$

derivate

$$\ln(y) = \ln(\sin(x)) + 3x \cdot \ln(x^2+x) - \ln(110)$$

$$\frac{y'}{y} = \frac{1}{\sin(x)} \cdot \cos(x) + 3 \cdot \ln(x^2+x) + 3x \cdot \frac{1}{x^2+x} \cdot (2x+1) - 0$$

$$y' = y \cdot \left[\cot(x) + 3 \ln(x^2+x) + \frac{6x^2+3x}{x^2+x} \right]$$

$$y' = y \cdot \left[\cot(x) + 3 \ln(x^2+x) + \frac{6x^2+3x}{x^2+x} \right] \cdot \left[\frac{\sin(x) \cdot (x^2+x)^{3x}}{110} \right]$$

Pregunta 3

) + 2

a) Sea $y^2 = x^3 + (\sin(x \cdot y) - x \cdot \ln(y))$. Determine $\frac{dy}{dx}$

2

$$2y \cdot y' = 3x^2 + \cos(xy) \cdot (y + x y') - (\ln(y) + x \cdot \frac{1}{y} \cdot y')$$

$$2y \cdot y' + \frac{x}{y} \cdot y' - \cos(xy) \cdot (x y') = 3x^2 - \ln(y) + \cos(xy) \cdot y$$

$$y' \left(2y + \frac{x}{y} - \cos(xy) \cdot (x) \right) = 3x^2 - \ln(y) + \cos(xy) \cdot y$$

$$y' = \frac{(3x^2 - \ln(y)) + \cos(xy) \cdot y}{\left(2y + \frac{x}{y} - \cos(xy) \cdot x \right)}$$

b) Si $y + x \cdot \sqrt{1+2y} = x^2$ Determine $y'(1,0)$

$$y' + \left(\sqrt{1+2y} + x \cdot \frac{1}{2\sqrt{1+2y}} \cdot 2y' \right) = 2x$$

$$y' + \frac{2xy'}{\sqrt{1+2y}} = 2x - \sqrt{1+2y}$$

$$y' \left(1 + \frac{x}{\sqrt{1+2y}} \right) = 2x - \sqrt{1+2y}$$

$$y' = \frac{2x - \sqrt{1+2y}}{1 + \frac{x}{\sqrt{1+2y}}}$$

$$y' = \frac{2(1) - \sqrt{1}}{1 + \frac{1}{\sqrt{1}}}$$

$$\boxed{y' \Big|_{(1,0)} = \frac{1}{2}}$$

Pregunta 4

(-2)+2

$$P_1) \vee P(1, -4)$$

i +2

$$3x^2 \cdot y + y^2 - 3x = 1$$

$$(6x \cdot y + 3x^2 \cdot y') + 2y \cdot y' - 3 = 0$$

$$3x^2 \cdot y' + 2y \cdot y' = 4 - 6x \cdot y$$

$$y' = \frac{3 - 6xy}{3x^2 + 2y}$$

$$y' = \frac{3 - 6(1)(-4)}{3(1)^2 + 2(-4)}$$

$$y' = \frac{3 + 24}{3 - 8}$$

$$m_T = \frac{27}{-5}$$

$$y' = \frac{27}{-5}$$

$$y - (-4) = \frac{-27}{5} (x - 1)$$

$$y = \frac{-27}{5} x + \frac{27}{5} - 4 \cdot 5$$

$$L_T = \boxed{y = \frac{-27}{5} x + \frac{7}{5}}$$

$$b) \quad m_r = 4$$

$$m_n = -\frac{1}{4}$$

$$\bullet \quad y' = 3x^2 - 8$$

$$4 \neq 3x^2 - 8$$

$$4 + 8 = 3x^2$$

$$\frac{12}{3} = x^2$$

$$4 = x^2$$

$$\boxed{2 \pm x}$$

$$y = x^3 - 8x + 2 \quad \vee \quad y = (-2)^3 - 8(-2) + 2$$

$$y = (2)^3 - 8(2) + 2 \quad \vee \quad y = -8 + 16 + 2$$

$$y = -6 \quad \vee \quad y = 10$$

$$1) P(2, -6)$$

$$2) P(-2, 10)$$

$$L_{n_1} = y - (-6) = -\frac{1}{4}(x - 2)$$

$$y + 6 = -\frac{1}{4}x + \frac{1}{2}$$

$$y = -\frac{1}{4}x - \frac{11}{2}$$

$$L_{n_2} = y - 10 = -\frac{1}{4}(x + 2)$$

$$y = -\frac{1}{4}x - \frac{1}{2} + 10$$

$$y = -\frac{1}{4}x + \frac{19}{2}$$