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Guía N°6

PARTE 1

Derive;

1. $f(x) = 4$

2. $f(x) = -3$

3. $f(x) = \frac{1}{4}$

4. $f(x) = \sqrt{5}$

5. $f(x) = x$

6. $f(x) = x^2$

7. $f(x) = x^3$

8. $f(x) = x^4$

9. $f(x) = x^5$

10. $f(x) = x^6$

11. $f(x) = x^{-1}$

12. $f(x) = x^{-2}$

13. $f(x) = x^{-3}$

14. $f(x) = x^{\frac{1}{2}}$

15. $f(x) = x^{\frac{2}{5}}$

16. $f(x) = x^{-\frac{5}{4}}$

17. $f(x) = e^x$

18. $f(x) = 2^x$

19. $f(x) = 3^x$

20. $f(x) = 5^x$

21. $f(x) = \ln(x)$

22. $f(x) = \log_2(x)$

23. $f(x) = \log_3(x)$

24. $f(x) = \log_5(x)$

25. $f(x) = \sqrt{x}$

26. $f(x) = \sqrt[3]{x}$

27. $f(x) = \sqrt[4]{x}$

28. $f(x) = \sqrt[5]{x}$

29. $f(x) = \sqrt[6]{x}$

30. $f(x) = \sqrt[7]{x}$

PARTE 2

Derive

1. $y = 2x^3 + 4x^2 - 5x + 8$

2. $y = -5 + 3x - \frac{3}{2}x^2 - 7x^3$

3. $y = (x - 2)^4$

4. $y = (x^2 + 2)^3$

5. $y = (4 - x^2)^{10}$

6. $y = (2x^2 + 4x - 5)^6$

7. $y = \frac{1}{5}x^{\frac{5}{2}} + \frac{1}{3}x^{\frac{3}{2}}$

8. $y = (x^2 - 4)^{\frac{3}{2}}$

9. $y = (1 - x^2)^{\frac{1}{2}}$

10. $y = \frac{6}{x} + \frac{4}{x^2} - \frac{3}{x^3}$

11. $y = x^3(x + 1)^2$

12. $y = (x + 1)^3(x - 3)^2$

13. $y = (x + 2)^2(2 - x)^3$

14. $y = \frac{x + 1}{x - 1}$

15. $y = \frac{x^2 + 2x - 3}{x^2}$

16. $y = \frac{x^2 + 1}{x^2 + 2}$

17. $y = \frac{1}{(2x + 1)^3}$

18. $y = \frac{1}{(x^2 - 9)^{\frac{1}{2}}}$

$$19. \quad y = \frac{1}{(16 - x^2)^{\frac{1}{2}}}$$

$$20. \quad y = \frac{x}{(x + 1)^{\frac{1}{2}}}$$

$$22. \quad y = \frac{(x^2 + 2)^{\frac{1}{2}}}{x}$$

$$23. \quad y = e^{x^2+1}$$

$$24. \quad y = x^2 e^{-x^2}$$

$$25. \quad y = \frac{e^{2x}}{x^2}$$

$$26. \quad y = \sin(3x^2 + x)$$

$$27. \quad y = \tan(\sqrt{x} - x^9)$$

$$28. \quad y = \sec(e^x + x)$$

$$29. \quad y = \sin\left(\frac{2+x^2}{1-x^3}\right)$$

$$30. \quad y = (x^3 + 5)\cos(9x)$$

PARTE 3

Derive:

1. $f(x) = (x^3 + 4x - 3)^5$

2. $f(x) = (2x^5 - 5x - 2)^7$

3. $f(x) = \sqrt{x^4 - 3x + 6}$

4. $f(x) = \sqrt{x^5 + x + 1}$

5. $f(x) = e^{x^3+5x-2}$

6. $f(x) = e^{2x^5-3x^2+x-2}$

7. $f(x) = \ln(x^5 - 2x + 7)$

8. $f(x) = \ln(2x^4 - x + 5)$

9. $f(x) = \cos(x^3)$

10. $f(x) = \sin(x^6)$

11. $f(x) = \sin(e^{x^3})$

12. $f(x) = \sin^3(x^4)$

13. $f(x) = \sin^4(\cos^2(x^3))$

14. $f(x) = \ln(\sin(x^5))$

15. $f(x) = \sin(\ln(\sin(x)))$

16. $f(x) = (x^3 + x^2 + 4)^6$

17. $f(x) = \sin((x^4 + 2)^4)$

18. $f(x) = e^{x^3-4x}$

19. $f(x) = e^{\sin(x^2)}$

20. $f(x) = \cos^2(e^{x^2})$

21. $f(x) = \ln^3(\sin(e^{2x}))$

PARTE 4

En cada caso, determine $f'(x)$. Suponga que g es derivable en $\mathbb{R}$

1. $f(x) = g\left(\frac{1}{x^2}\right)$ donde $g'(x) = \frac{1}{x^4 + 1}$
2. $f(x) = g(\sin(x))$ donde $g'(x) = \frac{1}{x^2 + 1}$
3. $f(x) = g^2(x^2)$ donde $g'(x) = \frac{1}{x^2 + 2}$
4. $f(x) = g(g(x^3))$ donde $g'(x) = e^{x^2}$
5. $f(x) = g(g^2(2x))$ donde $g'(x) = \sin(x^2)$

Solución:

PARTE 1

1. $f(x) = 4$ $f'(x) = 0$
2. $f(x) = -3$ $f'(x) = 0$
3. $f(x) = \frac{1}{4}$ $f'(x) = 0$
4. $f(x) = \sqrt{5}$ $f'(x) = 0$
5. $f(x) = x$ $f'(x) = 1$
6. $f(x) = x^2$ $f'(x) = 2x$
7. $f(x) = x^3$ $f'(x) = 3x^2$
8. $f(x) = x^4$ $f'(x) = 4x^3$
9. $f(x) = x^5$ $f'(x) = 5x^4$
10. $f(x) = x^6$ $f'(x) = 6x^5$
11. $f(x) = x^{-1}$ $f'(x) = (-1)x^{-1-1} = -x^{-2}$
12. $f(x) = x^{-2}$ $f'(x) = -2x^{-2-1} = -2x^{-3}$
13. $f(x) = x^{-3}$ $f'(x) = -3x^{-3-1} = -3x^{-4}$
14. $f(x) = x^{\frac{1}{2}}$ $f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$
15. $f(x) = x^{\frac{2}{5}}$ $f'(x) = \frac{2}{5}x^{\frac{2}{5}-1} = \frac{2}{5}x^{-\frac{3}{5}}$
16. $f(x) = x^{-\frac{5}{4}}$ $f'(x) = -\frac{5}{4}x^{-\frac{5}{4}-1} = -\frac{5}{4}x^{-\frac{9}{4}}$
17. $f(x) = e^x$ $f'(x) = e^x$
18. $f(x) = 2^x$ $f'(x) = 2^x \ln(2)$
19. $f(x) = 3^x$ $f'(x) = 3^x \ln(3)$
20. $f(x) = 5^x$ $f'(x) = 5^x \ln(5)$
21. $f(x) = \ln(x)$ $f'(x) = \frac{1}{x}$
22. $f(x) = \log_2(x)$ $f'(x) = \frac{1}{x \ln(2)}$

$$23. \quad f(x) = \log_3(x) \qquad f'(x) = \frac{1}{x \ln(3)}$$

$$24. \quad f(x) = \log_5(x) \qquad f'(x) = \frac{1}{x \ln(5)}$$

$$25. \quad f(x) = \sqrt{x} \qquad f'(x) = \frac{1}{2\sqrt{x}}$$

$$26. \quad f(x) = \sqrt[3]{x} \qquad f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

$$27. \quad f(x) = \sqrt[4]{x} \qquad f'(x) = \frac{1}{4\sqrt[4]{x^3}}$$

$$28. \quad f(x) = \sqrt[5]{x} \qquad f'(x) = \frac{1}{5\sqrt[5]{x^4}}$$

$$29. \quad f(x) = \sqrt[6]{x} \qquad f'(x) = \frac{1}{6\sqrt[6]{x^5}}$$

$$30. \quad f(x) = \sqrt[7]{x} \qquad f'(x) = \frac{1}{7\sqrt[7]{x^6}}$$

PARTE 2

1. $y = 2x^3 + 4x^2 - 5x + 8$
 $y' = 6x^2 + 8x - 5$

2. $y = -5 + 3x - \frac{3}{2}x^2 - 7x^3$
 $y' = 3 - 3x - 21x^2$

3. $y = (x - 2)^4$
 $y' = 4(x - 2)^3$

4. $y = (x^2 + 2)^3$
 $y' = 3(x^2 + 2)^2 \cdot 2x = 6x(x^2 + 2)^2$

5. $y = (4 - x^2)^{10}$
 $y' = 10(4 - x^2)^9 \cdot (-2x) = -20x(4 - x^2)^9$

6. $y = (2x^2 + 4x - 5)^6$
 $y' = 6(2x^2 + 4x - 5)^5 \cdot (4x + 4) = 24(x + 1)(2x^2 + 4x - 5)^5$

7. $y = \frac{1}{5}x^{\frac{5}{2}} + \frac{1}{3}x^{\frac{3}{2}}$
 $y' = \frac{1}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}}(x + 1)$

8. $y = (x^2 - 4)^{\frac{3}{2}}$
 $y' = \frac{3}{2}(x^2 - 4)^{\frac{1}{2}} \cdot 2x = 3x(x^2 - 4)^{\frac{1}{2}}$

$$9. \quad y = (1 - x^2)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(1 - x^2)^{-\frac{1}{2}} \cdot (-2x) = -\frac{x}{(1 - x^2)^{\frac{1}{2}}}$$

$$10. \quad y = \frac{6}{x} + \frac{4}{x^2} - \frac{3}{x^3}$$

$$y = 6x^{-1} + 4x^{-2} - 3x^{-3}$$

$$y' = -6x^{-2} - 8x^{-3} + 9x^{-4} = -\frac{6}{x^2} - \frac{8}{x^3} + \frac{9}{x^4}$$

$$11. \quad y = x^3(x + 1)^2$$

$$y' = 3x^2(x + 1)^2 + x^3 \cdot 2(x + 1) = x^2(x + 1)(3(x + 1) + 2x)$$

$$y' = x^2(x + 1)(5x + 3)$$

$$12. \quad y = (x + 1)^3(x - 3)^2$$

$$y' = 3(x + 1)^2(x - 3)^2 + (x + 1)^3 \cdot 2(x - 3)$$

$$y' = (x + 1)^2(x - 3)(3(x - 3) + 2(x + 1))$$

$$y' = (x + 1)^2(x - 3)(5x - 7)$$

$$13. \quad y = (x + 2)^2(2 - x)^3$$

$$y' = 2(x + 2)(2 - x)^3 + (x + 2)^2 \cdot 3(2 - x)^2(-1)$$

$$y' = (x + 2)(2 - x)^2(2(2 - x) - 3(x + 2))$$

$$y' = (x + 2)(2 - x)^2(-5x - 2)$$

$$y' = -(x + 2)(2 - x)^2(5x + 2)$$

$$14. \quad y = \frac{x + 1}{x - 1}$$

$$y' = \frac{x - 1 - (x + 1)}{(x - 1)^2} = -\frac{2}{(x - 1)^2}$$

$$\begin{aligned}
 15. \quad y &= \frac{x^2 + 2x - 3}{x^2} \\
 y' &= \frac{(2x + 2)x^2 - (x^2 + 2x - 3) \cdot 2x}{x^4} \\
 y' &= \frac{x((2x + 2)x - (x^2 + 2x - 3) \cdot 2)}{x^4} \\
 y' &= \frac{2x^2 + 2x - 2x^2 - 4x + 6}{x^3} \\
 y' &= \frac{6 - 2x}{x^3}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad y &= \frac{x^2 + 1}{x^2 + 2} \\
 y' &= \frac{2x(x^2 + 2) - (x^2 + 1) \cdot 2x}{(x^2 + 2)^2} \\
 y' &= \frac{2x(x^2 + 2 - x^2 - 1)}{(x^2 + 2)^2} \\
 y' &= \frac{2x}{(x^2 + 2)^2}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad y &= \frac{1}{(2x + 1)^3} \\
 y &= (2x + 1)^{-3} \\
 y' &= -3(2x + 1)^{-4} \cdot 2 = -6(2x + 1)^{-4} = -\frac{6}{(2x + 1)^4}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad y &= \frac{1}{(x^2 - 9)^{\frac{1}{2}}} \\
 y &= (x^2 - 9)^{-\frac{1}{2}} \\
 y' &= -\frac{1}{2}(x^2 - 9)^{-\frac{3}{2}} \cdot 2x = -\frac{x}{(x^2 - 9)^{\frac{3}{2}}}
 \end{aligned}$$

$$19. \quad y = \frac{1}{(16 - x^2)^{\frac{1}{2}}}$$

$$y = (16 - x^2)^{-\frac{1}{2}}$$

$$y' = -\frac{1}{2}(16 - x^2)^{-\frac{3}{2}} \cdot (-2x) = \frac{x}{(16 - x^2)^{\frac{3}{2}}}$$

$$20. \quad y = \frac{x}{(x + 1)^{\frac{1}{2}}}$$

$$y' = \frac{(x + 1)^{\frac{1}{2}} - x \cdot \frac{1}{2}(x + 1)^{-\frac{1}{2}}}{x + 1}$$

$$y' = \frac{(x + 1)^{\frac{1}{2}} - x \cdot \frac{1}{2(x + 1)^{\frac{1}{2}}}}{x + 1}$$

$$y' = \frac{2x + 2 - x}{2(x + 1)^{\frac{1}{2}}(x + 1)}$$

$$y' = \frac{x + 2}{2(x + 1)^{\frac{3}{2}}}$$

$$22. \quad y = \frac{(x^2 + 2)^{\frac{1}{2}}}{x}$$

$$y' = \frac{\frac{1}{2}(x^2 + 2)^{-\frac{1}{2}} \cdot 2x \cdot x - (x^2 + 2)^{\frac{1}{2}}}{x^2}$$

$$y' = \frac{x^2(x^2 + 2)^{-\frac{1}{2}} - (x^2 + 2)^{\frac{1}{2}}}{x^2}$$

$$y' = \frac{\frac{x^2}{(x^2 + 2)^{\frac{1}{2}}} - (x^2 + 2)^{\frac{1}{2}}}{x^2}$$

$$y' = \frac{x^2 - x^2 - 2}{x^2(x^2 + 2)^{\frac{1}{2}}} = \frac{-2}{x^2(x^2 + 2)^{\frac{1}{2}}}$$

$$23. \quad y = e^{x^2+1}$$

$$y' = e^{x^2+1} \cdot 2x = 2xe^{x^2+1}$$

$$24. \quad y = x^2e^{-x^2}$$

$$y' = 2xe^{-x^2} + x^2e^{-x^2} \cdot -2x = 2xe^{-x^2} - 2x^3e^{-x^2}$$

$$y' = 2xe^{-x^2}(1 - x^2)$$

$$25. \quad y = \frac{e^{2x}}{x^2}$$

$$y' = \frac{e^{2x} \cdot 2 \cdot x^2 - e^{2x} \cdot 2x}{(x^2)^2} = \frac{2e^{2x}x^2 - 2xe^{2x}}{x^4}$$

$$y' = \frac{2xe^{2x}(x-1)}{x^4} = \frac{2e^{2x}(x-1)}{x^3}$$

$$26. \quad y = \sin(3x^2 + x)$$

$$y' = \cos(3x^2 + x) \cdot (6x + 1) = (6x + 1)\cos(3x^2 + x)$$

$$27. \quad y = \tan(\sqrt{x} - x^9)$$

$$y' = \sec^2(\sqrt{x} - x^9) \left(\frac{1}{2\sqrt{x}} - 9x^8 \right)$$

$$28. \quad y = \sec(e^x + x)$$

$$y' = \sec(e^x + x) \tan(e^x + x)(e^x + 1)$$

$$29. \quad y = \sin\left(\frac{2+x^2}{1-x^3}\right)$$

$$y' = \cos\left(\frac{2+x^2}{1-x^3}\right) \cdot \frac{2x(1-x^3) - (2+x^2) \cdot (-3x^2)}{(1-x^3)^2}$$

$$y' = \cos\left(\frac{2+x^2}{1-x^3}\right) \left(\frac{2x+x^4+6x^2}{(1-x^3)^2} \right)$$

$$30. \quad y = (x^3 + 5) \cos(9x)$$

$$y' = 3x^2 \cos(9x) + (x^3 + 5) \cdot (-\sin(9x)) \cdot 9$$

$$y' = 3x^2 \cos(9x) - 9(x^3 + 5) \sin(9x)$$

PARTE 3

$$1. \quad f(x) = (x^3 + 4x - 3)^5$$

$$f'(x) = 5(x^3 + 4x - 3)^4 \cdot (x^3 + 4x - 3)'$$

$$f'(x) = 5(x^3 + 4x - 3)^4 (3x^2 + 4)$$

$$2. \quad f(x) = (2x^5 - 5x - 2)^7$$

$$f'(x) = 7(2x^5 - 5x - 2)^6 \cdot (2x^5 - 5x - 2)'$$

$$f'(x) = 7(2x^5 - 5x - 2)^6 (10x^4 - 5)$$

$$3. \quad f(x) = \sqrt{x^4 - 3x + 6}$$

$$f'(x) = \frac{1}{2\sqrt{x^4 - 3x + 6}} \cdot (x^4 - 3x + 6)'$$

$$f'(x) = \frac{1}{2\sqrt{x^4 - 3x + 6}} \cdot (4x^3 - 3) = \frac{4x^3 - 3}{2\sqrt{x^4 - 3x + 6}}$$

$$4. \quad f(x) = \sqrt{x^5 + x + 1}$$

$$f'(x) = \frac{1}{2\sqrt{x^5 + x + 1}} \cdot (x^5 + x + 1)'$$

$$f'(x) = \frac{1}{2\sqrt{x^5 + x + 1}} \cdot (5x^4 + 1) = \frac{5x^4 + 1}{2\sqrt{x^5 + x + 1}}$$

$$5. \quad f(x) = e^{x^3 + 5x - 2}$$

$$f'(x) = e^{x^3 + 5x - 2} \cdot (x^3 + 5x - 2)'$$

$$f'(x) = e^{x^3 + 5x - 2} \cdot (3x^2 + 5) = e^{x^3 + 5x - 2} (3x^2 + 5)$$

$$6. \quad f(x) = e^{2x^5-3x^2+x-2}$$

$$f'(x) = e^{2x^5-3x^2+x-2} \cdot (2x^5 - 3x^2 + x - 2)'$$

$$f'(x) = e^{2x^5-3x^2+x-2} \cdot (10x^4 - 6x + 1) = e^{2x^5-3x^2+x-2}(10x^4 - 6x + 1)$$

$$7. \quad f(x) = \ln(x^5 - 2x + 7)$$

$$f'(x) = \frac{1}{x^5 - 2x + 7} \cdot (x^5 - 2x + 7)'$$

$$f'(x) = \frac{1}{x^5 - 2x + 7} \cdot (5x^4 - 2) = \frac{5x^4 - 2}{x^5 - 2x + 7}$$

$$8. \quad f(x) = \ln(2x^4 - x + 5)$$

$$f'(x) = \frac{1}{2x^4 - x + 5} \cdot (2x^4 - x + 5)'$$

$$f'(x) = \frac{1}{2x^4 - x + 5} \cdot (8x^3 - 1) = \frac{8x^3 - 1}{2x^4 - x + 5}$$

$$9. \quad f(x) = \cos(x^3)$$

$$f'(x) = -\sin(x^3) \cdot (x^3)'$$

$$f'(x) = -\sin(x^3) \cdot 3x^2 = -3x^2 \sin(x^3)$$

$$10. \quad f(x) = \sin(x^6)$$

$$f'(x) = \cos(x^6) \cdot (x^6)'$$

$$f'(x) = \cos(x^6) \cdot 6x^5 = 6x^5 \cos(x^6)$$

$$11. \quad f(x) = \sin(e^{x^3})$$

$$f'(x) = \cos(e^{x^3}) \cdot (e^{x^3})'$$

$$f'(x) = \cos(e^{x^3}) \cdot e^{x^3} \cdot (x^3)'$$

$$f'(x) = \cos(e^{x^3}) \cdot e^{x^3} \cdot 3x^2 = 3x^2 e^{x^3} \cos(e^{x^3})$$

$$12. \quad f(x) = \sin^3(x^4)$$

$$f'(x) = 3 \sin^2(x^4) \cdot (\sin(x^4))'$$

$$f'(x) = 3 \sin^2(x^4) \cdot \cos(x^4) \cdot (x^4)'$$

$$f'(x) = 3 \sin^2(x^4) \cdot \cos(x^4) \cdot 4x^3$$

$$f'(x) = 3 \sin^2(x^4) \cdot \cos(x^4) \cdot 4x^3 = 12 \sin^2(x^4) \cos(x^4) x^3$$

$$13. \quad f(x) = \sin^4(\cos^2(x^3))$$

$$f'(x) = 4 \sin^3(\cos^2(x^3)) \cdot (\sin(\cos^2(x^3)))'$$

$$f'(x) = 4 \sin^3(\cos^2(x^3)) \cdot \cos(\cos^2(x^3)) \cdot (\cos^2(x^3))'$$

$$f'(x) = 4 \sin^3(\cos^2(x^3)) \cdot \cos(\cos^2(x^3)) \cdot 2 \cos(x^3) \cdot (\cos(x^3))'$$

$$f'(x) = 4 \sin^3(\cos^2(x^3)) \cdot \cos(\cos^2(x^3)) \cdot 2 \cos(x^3) \cdot (-\sin(x^3)) \cdot (x^3)'$$

$$f'(x) = 4 \sin^3(\cos^2(x^3)) \cdot \cos(\cos^2(x^3)) \cdot 2 \cos(x^3) \cdot (-\sin(x^3)) \cdot 3x^2$$

$$f'(x) = -24x^2 \sin^3(\cos^2(x^3)) \cos(\cos^2(x^3)) \cos(x^3) \sin(x^3)$$

$$14. \quad f(x) = \ln(\sin(x^5))$$

$$f'(x) = \frac{1}{\sin(x^5)} \cdot (\sin(x^5))'$$

$$f'(x) = \frac{1}{\sin(x^5)} \cdot \cos(x^5) \cdot (x^5)'$$

$$f'(x) = \frac{1}{\sin(x^5)} \cdot \cos(x^5) \cdot 5x^4 = \frac{5x^4 \cos(x^5)}{\sin(x^5)}$$

$$15. \quad f(x) = \sin(\ln(\sin(x)))$$

$$f'(x) = \cos(\ln(\sin(x))) \cdot (\ln(\sin(x)))'$$

$$f'(x) = \cos(\ln(\sin(x))) \cdot \frac{1}{\sin(x)} \cdot (\sin(x))'$$

$$f'(x) = \cos(\ln(\sin(x))) \cdot \frac{1}{\sin(x)} \cdot \cos(x) = \frac{\cos(\ln(\sin(x))) \cos(x)}{\sin(x)}$$

$$16. \quad f(x) = (x^3 + x^2 + 4)^6$$

$$f'(x) = 6(x^3 + x^2 + 4)^5 \cdot (x^3 + x^2 + 4)'$$

$$f'(x) = 6(x^3 + x^2 + 4)^5 \cdot (3x^2 + 2x + 0) = 6(x^3 + x^2 + 4)^5 (3x^2 + 2x)$$

$$17. \quad f(x) = \sin((x^4 + 2)^4)$$

$$f'(x) = \cos((x^4 + 2)^4) \cdot ((x^4 + 2)^4)'$$

$$f'(x) = \cos((x^4 + 2)^4) \cdot 4(x^4 + 2)^3 \cdot (x^4 + 2)'$$

$$f'(x) = \cos((x^4 + 2)^4) \cdot 4(x^4 + 2)^3 \cdot (4x^3 + 0) = 16x^3(x^4 + 2)^3 \cos((x^4 + 2)^4)$$

$$18. \quad f(x) = e^{x^3 - 4x}$$

$$f'(x) = e^{x^3 - 4x} \cdot (x^3 - 4x)'$$

$$f'(x) = e^{x^3 - 4x} \cdot (3x^2 - 4) = (3x^2 - 4)e^{x^3 - 4x}$$

$$19. \quad f(x) = e^{\sin(x^2)}$$

$$f'(x) = e^{\sin(x^2)} \cdot (\sin(x^2))'$$

$$f'(x) = e^{\sin(x^2)} \cdot \cos(x^2) \cdot (x^2)'$$

$$f'(x) = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x = 2xe^{\sin(x^2)} \cos(x^2)$$

$$20. \quad f(x) = \cos^2(e^{x^2})$$

$$f'(x) = 2 \cos(e^{x^2}) \cdot (\cos(e^{x^2}))'$$

$$f'(x) = 2 \cos(e^{x^2}) \cdot (-\sin(e^{x^2})) \cdot (e^{x^2})'$$

$$f'(x) = 2 \cos(e^{x^2}) \cdot (-\sin(e^{x^2})) \cdot e^{x^2} \cdot (x^2)'$$

$$f'(x) = 2 \cos(e^{x^2}) \cdot (-\sin(e^{x^2})) \cdot e^{x^2} \cdot 2x$$

$$f'(x) = -4x \cos(e^{x^2}) \sin(e^{x^2}) e^{x^2}$$

$$21. \quad f(x) = \ln^3(\sin(e^{2x}))$$

$$f'(x) = 3 \ln^2(\sin(e^{2x})) \cdot (\ln(\sin(e^{2x})))'$$

$$f'(x) = 3 \ln^2(\sin(e^{2x})) \cdot \frac{1}{\sin(e^{2x})} \cdot (\sin(e^{2x}))'$$

$$f'(x) = 3 \ln^2(\sin(e^{2x})) \cdot \frac{1}{\sin(e^{2x})} \cdot \cos(e^{2x}) \cdot (e^{2x})'$$

$$f'(x) = 3 \ln^2(\sin(e^{2x})) \cdot \frac{1}{\sin(e^{2x})} \cdot \cos(e^{2x}) \cdot e^{2x} \cdot (2x)'$$

$$f'(x) = 3 \ln^2(\sin(e^{2x})) \cdot \frac{1}{\sin(e^{2x})} \cdot \cos(e^{2x}) \cdot e^{2x} \cdot 2$$

$$f'(x) = \frac{6 \ln^2(\sin(e^{2x})) \cos(e^{2x}) e^{2x}}{\sin(e^{2x})}$$

PARTE 4

1. $f(x) = g\left(\frac{1}{x^2}\right)$ donde $g'(x) = \frac{1}{x^4 + 1}$

$$f'(x) = g'\left(\frac{1}{x^2}\right) \cdot \left(\frac{1}{x^2}\right)'$$

$$f'(x) = \frac{1}{\left(\frac{1}{x^2}\right)^4 + 1} \cdot \left(-\frac{2}{x^3}\right)$$

$$f'(x) = \frac{x^8}{1 + x^8} \cdot \left(-\frac{2}{x^3}\right)$$

$$f'(x) = \frac{-2x^5}{1 + x^8}$$

2. $f(x) = g(\sin(x))$ donde $g'(x) = \frac{1}{x^2 + 1}$

$$f'(x) = g'(\sin(x)) \cdot (\sin(x))'$$

$$f'(x) = \frac{1}{(\sin(x))^2 + 1} \cdot \cos(x)$$

$$f'(x) = \frac{\cos(x)}{\sin^2(x) + 1}$$

3. $f(x) = g^2(x^2)$ donde $g'(x) = \frac{1}{x^2 + 2}$

$$f'(x) = 2g(x^2) \cdot g'(x^2) \cdot 2x$$

$$f'(x) = 2g(x^2) \cdot \frac{1}{(x^2)^2 + 2} \cdot 2x$$

$$f'(x) = \frac{4xg(x^2)}{x^4 + 2}$$

4. $f(x) = g(g(x^3))$ donde $g'(x) = e^{x^2}$

$$f'(x) = g'(g(x^3)) \cdot g'(x^3) \cdot 3x^2$$

$$f'(x) = e^{(g(x^3))^2} \cdot e^{(x^3)^2} \cdot 3x^2$$

$$f'(x) = 3x^2 e^{g^2(x^3)} e^{x^6}$$

$$f'(x) = 3x^2 e^{g^2(x^3) + x^6}$$

5. $f(x) = g(g^2(2x))$ donde $g'(x) = \sin(x^2)$

$$f'(x) = g'(g^2(2x)) \cdot 2g(2x) \cdot g'(2x) \cdot 2$$

$$f'(x) = \sin\left((g^2(2x))^2\right) \cdot 2g(2x) \cdot \sin\left((2x)^2\right) \cdot 2$$

$$f'(x) = 4g(2x) \sin(g^4(2x)) \sin(4x^2)$$