Guía 5 Límites

Sergio Yansen Núñez

Calcule los siguientes límites, si existen

1. $\lim_{x \to 9} \dfrac{\sqrt{x} - 3}{x^2 - 81}$

2. $\lim_{x \to 4} \dfrac{(\sqrt{x} - 2)(3x - 8)}{\sqrt[3]{x+4} - 2}$

3. $\lim_{x \to -8} \dfrac{\sqrt{1-x} - 3}{2 + \sqrt[3]{x}}$

4. $\lim_{x \to -1} \dfrac{x^3 + 4x^2 + 5x + 2}{3x^3 + 11x^2 + 13x + 5}$

5. $\lim_{x \to 2} \dfrac{(x^2 + x - 6)\left(1 - \sqrt{x-1}\right)}{x^2 - 4x + 4}$

6. $\lim_{x \to 0} \dfrac{\sqrt[3]{1+x^2} - \sqrt[4]{1-2x}}{x^3 + x}$

7. $\lim_{x \to 1} \dfrac{3x^2 - 5x + 2}{\sqrt{3x+1} - x^2 - 1}$

8. $\lim_{x \to 1} \dfrac{\sqrt[4]{x} - 1}{1 - \sqrt[3]{x}}$

9. $\lim_{x \to 2} \dfrac{\sqrt[3]{1-x} + 1}{\sqrt{x-1} - 1}$

10. $\lim_{x \to 2} \dfrac{(x^2 + x - 6)\left(1 - \sqrt{x-1}\right)}{(x-2)^2}$

11. $\lim_{x \to 4} \dfrac{(\sqrt{x} - 2)\left(\sqrt[3]{2x} - 2\right)}{\left(\sqrt[3]{x+4} - 2\right)(x^2 - 16)}$

12. $\lim\limits_{x \to 1} \dfrac{2x^4 - 2x^3 - 6x^2 + 10x - 4}{x^5 + 2x^4 - 3x^3 - x^2 - 2x + 3}$

13. $\lim\limits_{x\to 0} \dfrac{\tan(3x)}{x}$

14. $\lim\limits_{x\to 0} \dfrac{\sin(2x) + \sin(3x)}{\sin(4x) + \sin(5x)}$

15. $\lim\limits_{x\to 0} \dfrac{x + \sin^2(2x)}{x + \sin^2(3x)}$

16. $\lim\limits_{x\to 0} \dfrac{1 - \cos(ax)}{x}$, siendo a una constante real distinta de cero.

17. $\lim\limits_{x\to 0} \dfrac{1 - \cos(ax)}{x^2}$, siendo a una constante real distinta de cero.

18 $\lim\limits_{u\to 0} \dfrac{\sin(u + a) - \sin(a)}{u}$

19. $\lim\limits_{u\to 0} \dfrac{\sin(u + \pi)}{u}$

20. $\lim\limits_{u\to 0} \dfrac{\sin\left(2\left(u + \frac{1}{2}\right) - 1\right)}{4\left(u + \frac{1}{2}\right)^2 - 1}$

21. $\lim\limits_{u\to 0} \dfrac{\cos(u + \frac{\pi}{2})}{2\left(u + \frac{\pi}{2}\right) - \pi}$

22. $\lim\limits_{u\to 0} \dfrac{\sin\left(4\left(u + \frac{\pi}{4}\right)\right)}{4\left(u + \frac{\pi}{4}\right) - \pi}$

23. $\lim\limits_{u\to 0} \dfrac{\sin\left(2\left(u + \frac{\pi}{4}\right)\right) - 1}{4\left(u + \frac{\pi}{4}\right) - \pi}$

24. $\lim\limits_{x\to 0} \dfrac{1 - \sqrt{\cos(5x)}}{x}$

25. $\lim\limits_{u\to 0} \dfrac{\sin\left(u + \frac{\pi}{2}\right) - 1}{\cos\left(2\left(u + \frac{\pi}{2}\right)\right) + 1}$

26. $$\lim_{x \to 0} \frac{\sqrt{1+\sin(2x)} - \sqrt{1-\sin(3x)}}{x}$$

SOLUCIONES

Calcular los siguientes límites, si existen.

1. $$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x^2 - 81} = \lim_{x \to 9} \frac{\sqrt{x} - 3}{(x+9)(x-9)} \bullet \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$

$$= \lim_{x \to 9} \frac{x-9}{(x+9)(x-9)(\sqrt{x} + 3)}$$

$$= \lim_{x \to 9} \frac{1}{(x+9)(\sqrt{x} + 3)}$$

$$= \frac{1}{108}$$

2. $$\lim_{x \to 4} \frac{\sqrt{x} - 2}{\sqrt[3]{x+4} - 2}(3x-8)$$

$$= \lim_{x \to 4} \frac{\sqrt{x} - 2}{\sqrt[3]{x+4} - 2}(3x-8) \bullet \frac{\sqrt{x} + 2}{\sqrt{x} + 2} \bullet \frac{\left(\sqrt[3]{x+4}\right)^2 + 2\sqrt[3]{x+4} + 4}{\left(\sqrt[3]{x+4}\right)^2 + 2\sqrt[3]{x+4} + 4}$$

$$= \lim_{x \to 4} \frac{x-4}{x-4}(3x-8) \bullet \frac{\left(\sqrt[3]{x+4}\right)^2 + 2\sqrt[3]{x+4} + 4}{\sqrt{x} + 2}$$

$$= \lim_{x \to 4} (3x-8) \bullet \frac{\left(\sqrt[3]{x+4}\right)^2 + 2\sqrt[3]{x+4} + 4}{\sqrt{x} + 2}$$

$$=12$$

3. $$\lim_{x \to -8} \frac{\sqrt{1-x} - 3}{2 + \sqrt[3]{x}}$$

$$= \lim_{x \to -8} \frac{\sqrt{1-x} - 3}{2 + \sqrt[3]{x}} \bullet \frac{\sqrt{1-x} + 3}{\sqrt{1-x} + 3} \bullet \frac{4 - 2\sqrt[3]{x} + (\sqrt[3]{x})^2}{4 - 2\sqrt[3]{x} + (\sqrt[3]{x})^2}$$

$$= \lim_{x \to -8} \frac{-(8+x)}{8+x} \bullet \frac{4 - 2\sqrt[3]{x} + (\sqrt[3]{x})^2}{\sqrt{1-x} + 3}$$

$$= \lim_{x \to -8} \frac{-\left(4 - 2\sqrt[3]{x} + (\sqrt[3]{x})^2\right)}{\sqrt{1-x} + 3}$$

$= -2$

4. $$\lim_{x \to -1} \frac{x^3 + 4x^2 + 5x + 2}{3x^3 + 11x^2 + 13x + 5}$$

$$= \lim_{x \to -1} \frac{(x+2)(x+1)^2}{(3x+5)(x+1)^2}$$

$$= \lim_{x \to -1} \frac{x+2}{3x+5}$$

$= \frac{1}{2}$

5. $$\lim_{x \to 2} \frac{(x^2 + x - 6)\left(1 - \sqrt{x-1}\right)}{x^2 - 4x + 4}$$

$$= \lim_{x \to 2} \frac{(x+3)(x-2)\left(1 - \sqrt{x-1}\right)}{(x-2)^2} \bullet \frac{1 + \sqrt{x-1}}{1 + \sqrt{x-1}}$$

$$= \lim_{x \to 2} \frac{-(x+3)(x-2)(x-2)}{(x-2)^2} \bullet \frac{1}{1 + \sqrt{x-1}}$$

$$= \lim_{x \to 2} \frac{-(x+3)}{\left(1 + \sqrt{x-1}\right)}$$

$= -\frac{5}{2}$

6. $$\lim_{x \to 0} \frac{\sqrt[3]{1+x^2} - \sqrt[4]{1-2x}}{x^3 + x}$$

$$= \lim_{x \to 0} \frac{\sqrt[3]{1+x^2} - \sqrt[4]{1-2x}}{x(x^2+1)} \bullet \frac{\sqrt[3]{1+x^2} + \sqrt[4]{1-2x}}{\sqrt[3]{1+x^2} + \sqrt[4]{1-2x}}$$

$$= \lim_{x \to 0} \frac{\sqrt[3]{(1+x^2)^2} - \sqrt{1-2x}}{x(x^2+1)} \bullet \frac{1}{\sqrt[3]{1+x^2} + \sqrt[4]{1-2x}} \bullet \frac{\sqrt[3]{\left(1+x^2\right)^2}+\sqrt{1-2x}}{\sqrt[3]{\left(1+x^2\right)^2}+\sqrt{1-2x}}$$

$$= \lim_{x \to 0} \frac{\sqrt[3]{\left(1+x^2\right)^4}-(1-2x)}{x\left(x^2+1\right)\left(\sqrt[3]{1+x^2}+\sqrt[4]{1-2x}\right)\left(\sqrt[3]{\left(1+x^2\right)^2}+\sqrt{1-2x}\right)} \bullet \frac{\sqrt[3]{\left(1+x^2\right)^8}+\sqrt[3]{\left(1+x^2\right)^4}(1-2x)+(1-2x)^2}{\sqrt[3]{\left(1+x^2\right)^8}+\sqrt[3]{\left(1+x^2\right)^4}(1-2x)+(1-2x)^2}$$

$$= \lim_{x \to 0} \frac{\left(1+x^2\right)^4-(1-2x)^3}{x\left(x^2+1\right)\left(\sqrt[3]{1+x^2}+\sqrt[4]{1-2x}\right)\left(\sqrt[3]{\left(1+x^2\right)^2}+\sqrt{1-2x}\right)} \bullet \frac{1}{\sqrt[3]{\left(1+x^2\right)^8}+\sqrt[3]{\left(1+x^2\right)^4}(1-2x)+(1-2x)^2}$$

$$= \lim_{x \to 0} \frac{x\left(-8x+6x^3+4x^5+x^7+6+8x^2\right)}{x\left(x^2+1\right)\left(\sqrt[3]{1+x^2}+\sqrt[4]{1-2x}\right)\left(\sqrt[3]{\left(1+x^2\right)^2}+\sqrt{1-2x}\right)} \bullet \frac{1}{\sqrt[3]{\left(1+x^2\right)^8}+\sqrt[3]{\left(1+x^2\right)^4}(1-2x)+(1-2x)^2}$$

$$= \lim_{x \to 0} \frac{-8x+6x^3+4x^5+x^7+6+8x^2}{\left(x^2+1\right)\left(\sqrt[3]{1+x^2}+\sqrt[4]{1-2x}\right)\left(\sqrt[3]{\left(1+x^2\right)^2}+\sqrt{1-2x}\right)} \bullet \frac{1}{\sqrt[3]{\left(1+x^2\right)^8}+\sqrt[3]{\left(1+x^2\right)^4}(1-2x)+(1-2x)^2}$$

$$=\frac{1}{2}$$

7. $$\lim_{x \to 1} \frac{3x^2 - 5x + 2}{\sqrt{3x+1} - x^2 - 1}$$

$$= \lim_{x \to 1} \frac{(3x-2)(x-1)}{\sqrt{3x+1} - x^2 - 1} \bullet \frac{\sqrt{3x+1} + x^2 + 1}{\sqrt{3x+1} + x^2 + 1}$$

$$= \lim_{x \to 1} \frac{(3x-2)(x-1)\left(\sqrt{3x+1} + x^2 + 1\right)}{3x + 1 - (x^2+1)^2}$$

$$= \lim_{x \to 1} \frac{(3x-2)(x-1)\left(\sqrt{3x+1} + x^2 + 1\right)}{-x(x-1)(x^2+x+3)}$$

$$= \lim_{x \to 1} \frac{(3x-2)\left(\sqrt{3x+1} + x^2 + 1\right)}{-x(x^2+x+3)}$$

$$=-\frac{4}{5}$$

8. $\lim\limits_{x \to 1} \dfrac{\sqrt[4]{x} - 1}{1 - \sqrt[3]{x}}$

$= \lim\limits_{x \to 1} \dfrac{\sqrt[4]{x} - 1}{1 - \sqrt[3]{x}} \bullet \dfrac{\sqrt[4]{x} + 1}{\sqrt[4]{x} + 1} \bullet \dfrac{1 + \sqrt[3]{x} + \sqrt[3]{x^2}}{1 + \sqrt[3]{x} + \sqrt[3]{x^2}}$

$= \lim\limits_{x \to 1} \dfrac{\sqrt{x} - 1}{1 - x} \bullet \dfrac{1 + \sqrt[3]{x} + \sqrt[3]{x^2}}{\sqrt[4]{x} + 1} \bullet \dfrac{\sqrt{x} + 1}{\sqrt{x} + 1}$

$= \lim\limits_{x \to 1} \dfrac{x - 1}{-(x - 1)} \bullet \dfrac{1 + \sqrt[3]{x} + \sqrt[3]{x^2}}{\sqrt[4]{x} + 1} \bullet \dfrac{1}{\sqrt{x} + 1}$

$= \lim\limits_{x \to 1} \dfrac{-\left(1 + \sqrt[3]{x} + \sqrt[3]{x^2}\right)}{\sqrt[4]{x} + 1} \bullet \dfrac{1}{\sqrt{x} + 1}$

$= -\dfrac{3}{4}$

9. $\lim\limits_{x \to 2} \dfrac{\sqrt[3]{1 - x} + 1}{\sqrt{x - 1} - 1}$

$= \lim\limits_{x \to 2} \dfrac{\sqrt[3]{1 - x} + 1}{\sqrt{x - 1} - 1} \bullet \dfrac{\sqrt{x - 1} + 1}{\sqrt{x - 1} + 1} \bullet \dfrac{\left(\sqrt[3]{1-x}\right)^2 - \sqrt[3]{1-x} + 1}{\left(\sqrt[3]{1-x}\right)^2 - \sqrt[3]{1-x} + 1}$

$= \lim\limits_{x \to 2} \dfrac{2 - x}{x - 2} \bullet \dfrac{\sqrt{x - 1} + 1}{\left(\sqrt[3]{1 - x}\right)^2 - \sqrt[3]{1 - x} + 1}$

$= \lim\limits_{x \to 2} \dfrac{-\left(\sqrt{x - 1} + 1\right)}{\left(\sqrt[3]{1 - x}\right)^2 - \sqrt[3]{1 - x} + 1}$

$= -\dfrac{2}{3}$

10. $\lim\limits_{x \to 2} \dfrac{(x^2 + x - 6)\left(1 - \sqrt{x - 1}\right)}{(x - 2)^2}$

$= \lim\limits_{x \to 2} \dfrac{(x + 3)(x - 2)\left(1 - \sqrt{x - 1}\right)}{(x - 2)^2} \bullet \dfrac{1 + \sqrt{x - 1}}{1 + \sqrt{x - 1}}$

$$= \lim_{x \to 2} \frac{(x+3)(x-2)(2-x)}{(x-2)^2\left(1+\sqrt{x-1}\right)}$$

$$= \lim_{x \to 2} \frac{-(x+3)}{\left(1+\sqrt{x-1}\right)}$$

$$=-\frac{5}{2}$$

11. $$\lim_{x \to 4} \frac{\sqrt{x}-2}{\sqrt[3]{x+4}-2} \bullet \frac{\sqrt[3]{2x}-2}{x^2-16}$$

$$= \lim_{x \to 4} \frac{\sqrt{x}-2}{\sqrt[3]{x+4}-2} \bullet \frac{\sqrt[3]{2x}-2}{x^2-16} \bullet \frac{\sqrt{x}+2}{\sqrt{x}+2} \bullet \frac{\left(\sqrt[3]{x+4}\right)^2+2\sqrt[3]{x+4}+4}{\left(\sqrt[3]{x+4}\right)^2+2\sqrt[3]{x+4}+4} \bullet \frac{\left(\sqrt[3]{2x}\right)^2+2\sqrt[3]{2x}+4}{\left(\sqrt[3]{2x}\right)^2+2\sqrt[3]{2x}+4}$$

$$= \lim_{x \to 4} \frac{x-4}{x-4} \bullet \frac{2(x-4)}{(x+4)(x-4)} \bullet \frac{\left(\sqrt[3]{x+4}\right)^2+2\sqrt[3]{x+4}+4}{(\sqrt{x}+2)\left(\left(\sqrt[3]{2x}\right)^2+2\sqrt[3]{2x}+4\right)}$$

$$= \lim_{x \to 4} \frac{2}{x+4} \bullet \frac{\left(\sqrt[3]{x+4}\right)^2+2\sqrt[3]{x+4}+4}{(\sqrt{x}+2)\left(\left(\sqrt[3]{2x}\right)^2+2\sqrt[3]{2x}+4\right)}$$

$$=\frac{1}{16}$$

12. $$\lim_{x \to 1} \frac{2x^4-2x^3-6x^2+10x-4}{x^5+2x^4-3x^3-x^2-2x+3}$$

$$= \lim_{x \to 1} \frac{2(x+2)(x-1)^3}{(x+3)(x^2+x+1)(x-1)^2}:$$

$$= \lim_{x \to 1} \frac{2(x+2)(x-1)}{(x+3)(x^2+x+1)}:$$

$$=0$$

13. $$\lim_{x\to 0} \frac{\tan(3x)}{x} = \lim_{x\to 0} \frac{\frac{\sin(3x)}{\cos(3x)}}{x} = \lim_{x\to 0} \frac{\sin(3x)}{x\cos(3x)}$$

$$=\lim_{x\to0}\frac{\sin(3x)}{x}\bullet\frac{1}{\cos(3x)}=\lim_{x\to0}3\bullet\frac{\sin(3x)}{3x}\bullet\frac{1}{\cos(3x)}$$

$$=3\bullet1\bullet\frac{1}{1}=3$$

14. $$\lim_{x\to0}\frac{\sin(2x)+\sin(3x)}{\sin(4x)+\sin(5x)}=\lim_{x\to0}\frac{\sin(2x)+\sin(3x)}{\sin(4x)+\sin(5x)}\bullet\frac{\frac{1}{x}}{\frac{1}{x}}$$

$$=\lim_{x\to0}\frac{\frac{\sin(2x)}{x}+\frac{\sin(3x)}{x}}{\frac{\sin(4x)}{x}+\frac{\sin(5x)}{x}}$$

$$=\lim_{x\to0}\frac{2\bullet\underbrace{\frac{\sin(2x)}{2x}}+3\bullet\underbrace{\frac{\sin(3x)}{3x}}}{4\bullet\underbrace{\frac{\sin(4x)}{4x}}+5\bullet\underbrace{\frac{\sin(5x)}{5x}}}=\frac{2\bullet1+3\bullet1}{4\bullet1+5\bullet1}=\frac{5}{9}$$

15. $$\lim_{x\to0}\frac{x+\sin^2(2x)}{x+\sin^2(3x)}=\lim_{x\to0}\frac{x+\sin^2(2x)}{x+\sin^2(3x)}\bullet\frac{\frac{1}{x}}{\frac{1}{x}}$$

$$=\lim_{x\to0}\frac{1+\frac{\sin^2(2x)}{x}}{1+\frac{\sin^2(3x)}{x}}$$

$$=\lim_{x\to0}\frac{1+\frac{\sin(2x)}{x}\bullet\sin(2x)}{1+\frac{\sin(3x)}{x}\bullet\sin(3x)}=\lim_{x\to0}\frac{1+2\bullet\underbrace{\frac{\sin(2x)}{2x}}\bullet\sin(2x)}{1+3\bullet\underbrace{\frac{\sin(3x)}{3x}}\bullet\sin(3x)}$$

$$=\frac{1+2\bullet1\bullet0}{1+3\bullet1\bullet0}=1$$

16. $$\lim_{x\to0}\frac{1-\cos(ax)}{x}=\lim_{x\to0}\frac{1-\cos(ax)}{x}\bullet\frac{1+\cos(ax)}{1+\cos(ax)}$$

$$=\lim_{x\to0}\frac{1-\cos^2(ax)}{x}\bullet\frac{1}{1+\cos(ax)}=\lim_{x\to0}\frac{\sin^2(ax)}{x}\bullet\frac{1}{1+\cos(ax)}$$

$$=\lim_{x\to0}\frac{\sin(ax)}{x}\bullet\frac{\sin(ax)}{1+\cos(ax)}=\lim_{x\to0}a\bullet\frac{\sin(ax)}{ax}\bullet\frac{\sin(ax)}{1+\cos(ax)}$$

$$=a\bullet1\bullet\frac{0}{1+1}=0$$

17. $$\lim_{x\to0}\frac{1-\cos(ax)}{x^2}=\lim_{x\to0}\frac{1-\cos(ax)}{x^2}\bullet\frac{1+\cos(ax)}{1+\cos(ax)}$$

$$=\lim_{x\to 0} \frac{1-\cos^2(ax)}{x^2} \bullet \frac{1}{1+\cos(ax)}$$

$$=\lim_{x\to 0} \frac{\sin^2(ax)}{x^2} \bullet \frac{1}{1+\cos(ax)}$$

$$=\lim_{x\to 0} \frac{\sin(ax)}{x} \bullet \frac{\sin(ax)}{x} \bullet \frac{1}{1+\cos(ax)}$$

$$=\lim_{x\to 0} a \bullet \frac{\sin(ax)}{ax} \bullet a \bullet \frac{\sin(ax)}{ax} \bullet \frac{1}{1+\cos(ax)}$$

$$= a \bullet 1 \bullet a \bullet 1 \bullet \frac{1}{1+1} = \frac{a^2}{2}$$

6. $\lim_{x\to a} \frac{\sin(x)-\sin(a)}{x-a}$

18. $\lim_{u\to 0} \frac{\sin(u+a)-\sin(a)}{u}$

$$=\lim_{u\to 0} \frac{\sin(u)\cos(a)+\cos(u)\sin(a)-\sin(a)}{u}$$

$$=\lim_{u\to 0} \left(\frac{\sin(u)\cos(a)}{u} + \frac{\cos(u)\sin(a)-\sin(a)}{u} \right)$$

$$=\lim_{u\to 0} \left(\cos(a)\frac{\sin(u)}{u} + \sin(a)\frac{\cos(u)-1}{u} \right)$$

$$=\lim_{u\to 0} \left(\cos(a)\frac{\sin(u)}{u} - \sin(a)\frac{1-\cos(u)}{u} \right)$$

$\lim_{u\to 0} \frac{\sin(u)}{u} = 1$ y $\lim_{u\to 0} \frac{1-\cos(u)}{u} = 0$

(en la actividad 16 se obtuvo $\lim_{x\to 0} \frac{1-\cos(ax)}{x} = 0$)

Por lo tanto,

$$\lim_{u\to 0} \left(\cos(a)\frac{\sin(u)}{u} - \sin(a)\frac{1-\cos(u)}{u} \right) = \cos(a) \bullet 1 - \sin(a) \bullet 0 = \cos(a)$$

19. $\lim_{u\to 0} \frac{\sin(u+\pi)}{u}$

$$=\lim_{u\to 0} \frac{-\sin u}{u} = \lim_{u\to 0} -1 \bullet \frac{\sin u}{u} = -1 \bullet 1 = -1$$

20. $\lim\limits_{u\to 0} \dfrac{\sin\left(2\left(u+\frac{1}{2}\right)-1\right)}{4\left(u+\frac{1}{2}\right)^2-1}$

$=\lim\limits_{u\to 0} \dfrac{\sin(2u+1-1)}{4\left(u^2+u+\frac{1}{4}\right)-1}$

$=\lim\limits_{u\to 0} \dfrac{\sin(2u)}{4u^2+4u} = \lim\limits_{u\to 0} \dfrac{\sin(2u)}{4u(u+1)}$

$=\lim\limits_{u\to 0} \dfrac{\sin(2u)}{2u} \bullet \dfrac{1}{2(u+1)}$

$= 1 \bullet \dfrac{1}{2(0+1)} = \dfrac{1}{2}$

21. $\lim\limits_{u\to 0} \dfrac{\cos(u+\frac{\pi}{2})}{2\left(u+\frac{\pi}{2}\right)-\pi}$

$=\lim\limits_{u\to 0} \dfrac{\cos(u)\cos\left(\frac{\pi}{2}\right)-\sin(u)\sin\left(\frac{\pi}{2}\right)}{2u}$

$=\lim\limits_{u\to 0} \dfrac{-\sin(u)}{2u} = \lim\limits_{u\to 0} -\dfrac{1}{2} \bullet \dfrac{\sin(u)}{u} = -\dfrac{1}{2} \bullet 1 = -\dfrac{1}{2}$

22. $\lim\limits_{u\to 0} \dfrac{\sin\left(4\left(u+\frac{\pi}{4}\right)\right)}{4\left(u+\frac{\pi}{4}\right)-\pi}$

$=\lim\limits_{u\to 0} \dfrac{\sin(4u+\pi)}{4u}$

$=\lim\limits_{u\to 0} \dfrac{\sin(4u)\cos(\pi)+\cos(4u)\sin(\pi)}{4u}$

$\lim\limits_{u\to 0} \dfrac{-\sin(4u)}{4u} = \lim\limits_{u\to 0} -1 \bullet \dfrac{\sin(4u)}{4u} = -1 \bullet 1 = -1$

23. $\lim\limits_{u\to 0} \dfrac{\sin\left(2\left(u+\frac{\pi}{4}\right)\right)-1}{4\left(u+\frac{\pi}{4}\right)-\pi}$

$=\lim\limits_{u\to 0} \dfrac{\sin\left(2u+\frac{\pi}{2}\right)-1}{4u}$

$$=\lim_{u\to 0} \frac{\sin(2u)\cos\left(\frac{\pi}{2}\right) + \cos(2u)\sin\left(\frac{\pi}{2}\right) - 1}{4u}$$

$$=\lim_{u\to 0} \frac{\cos(2u) - 1}{4u}$$

$$=\lim_{u\to 0} -\frac{1}{4} \bullet \frac{1 - \cos(2u)}{u}$$

$$= -\frac{1}{4} \bullet \underbrace{\lim_{u\to 0} \frac{1 - \cos(2u)}{u}}_{\text{en la actividad 16 se obtuvo } \lim_{x\to 0} \frac{1-\cos(ax)}{x} = 0}$$

$$= -\frac{1}{4} \bullet 0 = 0$$

24. $$\lim_{x\to 0} \frac{1 - \sqrt{\cos(5x)}}{x} = \lim_{x\to 0} \frac{1 - \sqrt{\cos(5x)}}{x} \bullet \frac{1 + \sqrt{\cos(5x)}}{1 + \sqrt{\cos(5x)}}$$

$$=\lim_{x\to 0} \frac{1^2 - \left(\sqrt{\cos(5x)}\right)^2}{x} \bullet \frac{1}{1 + \sqrt{\cos(5x)}}$$

$$= \underbrace{\lim_{x\to 0} \frac{1 - \cos(5x)}{x}}_{\text{en la actividad 16 se obtuvo } \lim_{x\to 0} \frac{1-\cos(ax)}{x} = 0} \bullet \lim_{x\to 0} \frac{1}{1 + \sqrt{\cos(5x)}} = 0 \bullet \frac{1}{1+1}$$

$$= 0$$

25. $$\lim_{u\to 0} \frac{\sin\left(u + \frac{\pi}{2}\right) - 1}{\cos\left(2\left(u + \frac{\pi}{2}\right)\right) + 1}$$

$$=\lim_{u\to 0} \frac{\sin(u)\cos\left(\frac{\pi}{2}\right) + \cos(u)\sin\left(\frac{\pi}{2}\right) - 1}{\cos(2u + \pi) + 1}$$

$$=\lim_{u\to 0} \frac{\cos(u) - 1}{\cos(2u)\cos(\pi) - \sin(2u)\sin(\pi) + 1} = \lim_{u\to 0} \frac{\cos(u) - 1}{-\cos(2u) + 1}$$

$$=\lim_{u\to 0} \frac{\cos(u) - 1}{1 - \cos(2u)} \bullet \frac{\cos(u) + 1}{\cos(u) + 1} \bullet \frac{1 + \cos(2u)}{1 + \cos(2u)}$$

$$=\lim_{u\to 0} \frac{(\cos(u) - 1)(\cos(u) + 1)}{(1 - \cos(2u))(1 + \cos(2u))} \bullet \frac{1 + \cos(2u)}{\cos(u) + 1}$$

$$=\lim_{u\to 0} \frac{\cos^2(u)-1}{1-\cos^2(2u)} \bullet \frac{1+\cos(2u)}{\cos(u)+1}$$

$$=\lim_{u\to 0} \frac{-\sin^2(u)}{\sin^2(2u)} \bullet \frac{1+\cos(2u)}{\cos(u)+1}$$

$$=\lim_{u\to 0} \frac{\sin^2(u)}{\sin^2(2u)} \bullet \frac{-1-\cos(2u)}{\cos(u)+1}$$

Como $\lim_{u\to 0} \frac{-1-\cos(2u)}{\cos(u)+1} = \frac{-1-1}{1+1} = -1$, entonces se analizará $\lim_{u\to 0} \frac{\sin^2(u)}{\sin^2(2u)}$

$$\lim_{u\to 0} \frac{\sin^2(u)}{\sin^2(2u)} = \lim_{u\to 0} \frac{\sin(u)}{\sin(2u)} \bullet \frac{\sin(u)}{\sin(2u)} \bullet \frac{\frac{1}{u^2}}{\frac{1}{u^2}} = \lim_{u\to 0} \frac{\frac{\sin(u)}{u}}{\frac{\sin(2u)}{u}} \bullet \frac{\frac{\sin(u)}{u}}{\frac{\sin(2u)}{u}}$$

$$=\lim_{u\to 0} \frac{\frac{\sin(u)}{u}}{2\bullet\frac{\sin(2u)}{2u}} \bullet \frac{\frac{\sin(u)}{u}}{2\bullet\frac{\sin(2u)}{2u}} = \frac{1}{2\bullet 1} \bullet \frac{1}{2\bullet 1} = \frac{1}{4}$$

Por lo tanto,

$$\lim_{u\to 0} \frac{\sin^2(u)}{\sin^2(2u)} \bullet \frac{-1-\cos(2u)}{\cos(u)+1} = \frac{1}{4} \bullet (-1) = -\frac{1}{4}$$

26. $\lim_{x\to 0} \frac{\sqrt{1+\sin(2x)} - \sqrt{1-\sin(3x)}}{x}$

$$=\lim_{x\to 0} \frac{\sqrt{1+\sin(2x)} - \sqrt{1-\sin(3x)}}{x} \bullet \frac{\sqrt{1+\sin(2x)} + \sqrt{1-\sin(3x)}}{\sqrt{1+\sin(2x)} + \sqrt{1-\sin(2x)}}$$

$$=\lim_{x\to 0} \frac{\left(\sqrt{1+\sin(2x)}\right)^2 - \left(\sqrt{1-\sin(3x)}\right)^2}{x} \bullet \frac{1}{\sqrt{1+\sin(2x)} + \sqrt{1-\sin(3x)}}$$

$$=\lim_{x\to 0} \frac{1+\sin(2x)-(1-\sin(3x))}{x} \bullet \frac{1}{\sqrt{1+\sin(2x)} + \sqrt{1-\sin(3x)}}$$

$$=\lim_{x\to 0} \frac{1+\sin(2x)-1+\sin(3x)}{x} \bullet \frac{1}{\sqrt{1+\sin(2x)} + \sqrt{1-\sin(3x)}}$$

$$=\lim_{x\to 0} \frac{\sin(2x)+\sin(3x)}{x} \bullet \frac{1}{\sqrt{1+\sin(2x)} + \sqrt{1-\sin(3x)}}$$

$$=\lim_{x\to 0} \left(\frac{\sin(2x)}{x} + \frac{\sin(3x)}{x}\right) \bullet \frac{1}{\sqrt{1+\sin(2x)} + \sqrt{1-\sin(3x)}}$$

$$=\lim_{x\to 0}\left(2\bullet\frac{\sin(2x)}{2x}+3\bullet\frac{\sin(3x)}{3x}\right)\bullet\frac{1}{\sqrt{1+\sin(2x)}+\sqrt{1-\sin(3x)}}$$

$$=(2\bullet 1+3\bullet 1)\bullet\frac{1}{1+1}=\frac{5}{2}$$