

Guía 3

Sergio Yansen Núñez

1. Sean $f :]2, 8[\rightarrow \mathbb{R} \quad f(x) = 2x - 5$

$g :]1, 9[\rightarrow \mathbb{R} \quad g(x) = 3x - 1$

Determine explícitamente $g \circ f$

2. Sean $f : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = x^2$

$g :]-\infty, 9[\rightarrow \mathbb{R} \quad g(x) = 2x + 1$

Determine explícitamente $g \circ f$

3. Sean $f :]-4, +\infty[\rightarrow \mathbb{R} \quad f(x) = x^2 + 1$

$g :]5, +\infty[\rightarrow \mathbb{R} \quad g(x) = 4x + 1$

Determine explícitamente $g \circ f$

4. Sean $f : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = |x - 2|$

$g :]-\infty, 1[\rightarrow \mathbb{R} \quad g(x) = 2x$

Determine explícitamente $g \circ f$

5. Sean $f : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = |4x - 3|$

$g :]-\infty, 2] \rightarrow \mathbb{R} \quad g(x) = 3x + 1$

Determine explícitamente $g \circ f$

6. Sean $f : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = |x - 3|$

$g : [1, +\infty[\rightarrow \mathbb{R} \quad g(x) = 4x + 1$

Determine explícitamente $g \circ f$

7. Sean $f : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = |x + 4|$

$g :]1, +\infty[\rightarrow \mathbb{R} \quad g(x) = 7x - 1$

Determine explícitamente $g \circ f$

8. Sean $f : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = ||x + 3| - 2|$

$g :]-\infty, 3] \rightarrow \mathbb{R} \quad g(x) = 8x - 1$

Determine explícitamente $g \circ f$

SOLUCIÓN

1. Sean $f:]2, 8[\rightarrow \mathbb{R} \quad f(x) = 2x - 5$

$g:]1, 9[\rightarrow \mathbb{R} \quad g(x) = 3x - 1$

Determine explícitamente $g \circ f$

Solución:

$$\text{Dom}(g \circ f) = \{x \in \mathbb{R} / x \in \text{Dom}f \wedge f(x) \in \text{Dom}g\}$$

$$x \in]2, 8[\wedge 2x - 5 \in]1, 9[$$

$$2 < x < 8 \wedge 1 < 2x - 5 < 9$$

$$2 < x < 8 \wedge 6 < 2x < 14$$

$$2 < x < 8 \wedge 3 < x < 7$$

$$\Leftrightarrow 2 < x < 7$$

$$(g \circ f)(x) = 3(2x - 5) - 1 = 6x - 16$$

$$g \circ f:]2, 7[\rightarrow \mathbb{R} \quad (g \circ f)(x) = 6x - 16$$

2. Sean $f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = x^2$

$g:]-\infty, 9[\rightarrow \mathbb{R} \quad g(x) = 2x + 1$

Determine explícitamente $g \circ f$

Solución:

$$\text{Dom}(g \circ f) = \{x \in \mathbb{R} / x \in \text{Dom}f \wedge f(x) \in \text{Dom}g\}$$

$$x \in \mathbb{R} \wedge x^2 \in]-\infty, 9[$$

$$x \in \mathbb{R} \wedge x^2 < 9$$

$$x \in \mathbb{R} \wedge x^2 - 9 < 0$$

$$x \in \mathbb{R} \wedge (x + 3)(x - 3) < 0$$

	$-\infty$	-3	-3	3	3	$+\infty$
$x+3$		$-$		$+$		$+$
$x-3$		$-$		$-$		$+$
$(x+3)(x-3)$		$+$		$-$		$+$

$$x \in \mathbb{R} \quad \wedge \quad -3 < x < 3 < 0$$

$$\Leftrightarrow -3 < x < 3$$

$$(g \circ f)(x) = 2x^2 + 1$$

$$g \circ f :]-3, 3[\longrightarrow \mathbb{R} \quad (g \circ f)(x) = 2x^2 + 1$$

3. Sean $f :]-4, +\infty[\longrightarrow \mathbb{R} \quad f(x) = x^2 + 1$

$$g :]5, +\infty[\longrightarrow \mathbb{R} \quad g(x) = 4x + 1$$

Determine explícitamente $g \circ f$

Solución:

$$\text{Dom}(g \circ f) = \{x \in \mathbb{R} / x \in \text{Dom}f \quad \wedge \quad f(x) \in \text{Dom}g\}$$

$$x \in]-4, +\infty[\quad \wedge \quad x^2 + 1 \in]5, +\infty[$$

$$x \in]-4, +\infty[\quad \wedge \quad x^2 + 1 > 5$$

$$x \in]-4, +\infty[\quad \wedge \quad x^2 - 4 > 0$$

$$x \in]-4, +\infty[\quad \wedge \quad (x+2)(x-2) > 0$$

	$-\infty$	-2	-2	2	2	$+\infty$
$x+2$		$-$		$+$		$+$
$x-2$		$-$		$-$		$+$
$(x+2)(x-2)$		$+$		$-$		$+$

$$x \in]-4, +\infty[\quad \wedge \quad x \in]-\infty, -2[\cup]2, +\infty[$$

$$\Leftrightarrow x \in]-4, -2[\cup]2, +\infty[$$

$$(g \circ f)(x) = 4(x^2 + 1) + 1 = 4x^2 + 5$$

$$g \circ f :]-4, -2[\cup]2, +\infty[\longrightarrow \mathbb{R} \quad (g \circ f)(x) = 4x^2 + 5$$

4. Sean $f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = |x - 2|$

$g:]-\infty, 1[\rightarrow \mathbb{R} \quad g(x) = 2x$

Determine explícitamente $g \circ f$

Solución:

$$\text{Dom}(g \circ f) = \{x \in \mathbb{R} / x \in \text{Dom}f \wedge f(x) \in \text{Dom}g\}$$

$$x \in \mathbb{R} \wedge |x - 2| \in]-\infty, 1[$$

$$x \in \mathbb{R} \wedge |x - 2| < 1$$

$$-1 < x - 2 < 1 \quad / + 2$$

$$1 < x < 3$$

$$x \in]1, 3[$$

$$(g \circ f)(x) = 2|x - 2|$$

$$g \circ f:]1, 3[\rightarrow \mathbb{R} \quad (g \circ f)(x) = 2|x - 2|$$

5. Sean $f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = |4x - 3|$

$g:]-\infty, 2] \rightarrow \mathbb{R} \quad g(x) = 3x + 1$

Determine explícitamente $g \circ f$

Solución:

$$\text{Dom}(g \circ f) = \{x \in \mathbb{R} / x \in \text{Dom}f \wedge f(x) \in \text{Dom}g\}$$

$$x \in \mathbb{R} \wedge |4x - 3| \in]-\infty, 2]$$

$$x \in \mathbb{R} \wedge |4x - 3| \leq 2$$

$$-2 \leq 4x - 3 \leq 2 \quad / + 3$$

$$1 \leq 4x \leq 5 \quad / \cdot \frac{1}{4}$$

$$\frac{1}{4} \leq x \leq \frac{5}{4}$$

$$x \in \left[\frac{1}{4}, \frac{5}{4} \right]$$

$$(g \circ f)(x) = 3|4x - 3| + 1$$

$$g \circ f : \left[\frac{1}{4}, \frac{5}{4} \right] \longrightarrow \mathbb{R} \quad (g \circ f)(x) = 3|4x - 3| + 1$$

6. Sean $f : \mathbb{R} \longrightarrow \mathbb{R} \quad f(x) = |x - 3|$

$$g : [1, +\infty[\longrightarrow \mathbb{R} \quad g(x) = 4x + 1$$

Determine explícitamente $g \circ f$

Solución:

$$\text{Dom}(g \circ f) = \{x \in \mathbb{R} / x \in \text{Dom}f \wedge f(x) \in \text{Dom}g\}$$

$$x \in \mathbb{R} \quad \wedge \quad |x - 3| \in [1, +\infty[$$

$$x \in \mathbb{R} \quad \wedge \quad |x - 3| \geq 1$$

$$x - 3 \leq -1 \quad \vee \quad x - 3 \geq 1$$

$$x \leq 2 \quad \vee \quad x \geq 4$$

$$x \in]-\infty, 2] \cup [4, +\infty[$$

$$(g \circ f)(x) = 4|x - 3| + 1$$

$$g \circ f :]-\infty, 2] \cup [4, +\infty[\longrightarrow \mathbb{R} \quad (g \circ f)(x) = 4|x - 3| + 1$$

7. Sean $f : \mathbb{R} \longrightarrow \mathbb{R} \quad f(x) = |x + 4|$

$$g :]1, +\infty[\longrightarrow \mathbb{R} \quad g(x) = 7x - 1$$

Determine explícitamente $g \circ f$

Solución:

$$\text{Dom}(g \circ f) = \{x \in \mathbb{R} / x \in \text{Dom}f \wedge f(x) \in \text{Dom}g\}$$

$$x \in \mathbb{R} \quad \wedge \quad |x + 4| \in]1, +\infty[$$

$$x \in \mathbb{R} \quad \wedge \quad |x + 4| > 1$$

$$x + 4 < -1 \quad \vee \quad x + 4 > 1$$

$$x < -5 \quad \vee \quad x > -3$$

$$x \in]-\infty, -5[\cup]-3, +\infty[$$

$$(g \circ f)(x) = 7|x+4| - 1$$

$$g \circ f :]-\infty, -5[\cup]-3, +\infty[\longrightarrow \mathbb{R} \quad (g \circ f)(x) = 7|x+4| - 1$$

$$8. \quad \text{Sean } f : \mathbb{R} \longrightarrow \mathbb{R} \quad f(x) = ||x+3| - 2|$$

$$g :]-\infty, 3] \longrightarrow \mathbb{R} \quad g(x) = 8x - 1$$

Determine explícitamente $g \circ f$

Solución:

$$\text{Dom}(g \circ f) = \{x \in \mathbb{R} / x \in \text{Dom}f \wedge f(x) \in \text{Dom}g\}$$

$$x \in \mathbb{R} \quad \wedge \quad ||x+3| - 2| \in]-\infty, 3]$$

$$x \in \mathbb{R} \quad \wedge \quad ||x+3| - 2| \leq 3$$

$$-3 \leq |x+3| - 2 \leq 3 \quad / + 2$$

$$-1 \leq |x+3| \leq 5$$

$$-1 \leq |x+3| \quad \wedge \quad |x+3| \leq 5$$

$$\underbrace{-1}_{<0} \leq \underbrace{|x+3|}_{\geq 0} \quad \Leftrightarrow \quad x \in \mathbb{R}$$

$$|x+3| \leq 5 \quad \Leftrightarrow \quad -5 \leq x+3 \leq 5 \quad \Leftrightarrow \quad -8 \leq x \leq 2$$

$$x \in [-8, 2]$$

$$(g \circ f)(x) = 8||x+3| - 2| - 1$$

$$g \circ f : [-8, 2] \longrightarrow \mathbb{R} \quad (g \circ f)(x) = 8||x+3| - 2| - 1$$