① $\int (4x^5 + 3\cos(x) - \frac{3}{x})\,dx = 4\int x^5\,dx + 3\int \cos(x)\,dx - 3\cdot\int \frac{1}{x}\,dx$

$= \frac{4\cdot x^6}{6} + 3\sin(x) - 3\cdot \ln|x| + C$

$= \frac{2x^6}{3} + 3\sin(x) - 3\cdot\ln|x| + C$, $C \in \mathbb{R}$

② $\int \sqrt{x}(3x + 5x^3)\,dx = \int (x^{1/2}\cdot 3x^1 + x^{1/2}\cdot 5x^3)\,dx$

$= 3\int x^{3/2}\,dx + 5\int x^{7/2}\,dx$

$= 3\cdot\frac{x^{5/2}}{5/2} + 5\cdot\frac{x^{9/2}}{9/2} + C$

$= \frac{6}{5}x^{5/2} + \frac{10}{9}x^{9/2} + C$, $C \in \mathbb{R}$

$\frac{3}{2} + 1 = 5/2$

$\frac{7}{2} + 1 = 9/2$

1. $\int x \ln(x) dx$

2. $\int x^n \ln(x) dx$

3. $\int (x-2)^3 \ln(x-2) dx$

4. $\int \frac{\ln(x^2)}{x^2} dx$

5. $\int \ln(2x) dx$

6. $\int x^3 \ln(3x) dx$

7. $\int x^3 \ln(x^3)\, dx$

guía: Integral-PARTES

ILATE

$u = \ln(x)$ | $dv = x\,dx$

$du = \frac{1}{x}\,dx$ | $v = \frac{x^2}{2}$

$$\int x \cdot \ln(x)\,dx = uv - \int v\,du$$

$$= \ln(x) \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx$$

$$= \frac{\ln(x)x^2}{2} - \frac{1}{2}\int x\,dx$$

$$= \frac{\ln(x) \cdot x^2}{2} - \frac{1}{2} \cdot \frac{x^2}{2} + C$$

$$= \frac{\ln(x) \cdot x^2}{2} - \frac{x^2}{4} + C \quad , C \in \mathbb{R}$$

1. $\int x \ln(x) dx$

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(2)

$u = \ln(x)$ | $dv = x^n dx$

$du = \frac{1}{x} dx$ | $v = \frac{x^{n+1}}{n+1}$

$$\int x^n \cdot \ln(x) dx = \frac{\ln(x) \cdot x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1} \cdot \frac{1}{x} dx$$

$\frac{1}{n+1}$ CONSTANTE

$$= \frac{\ln(x) \cdot x^{n+1}}{n+1} - \frac{1}{n+1} \int x^n dx$$

$$= \frac{\ln(x) \cdot x^{n+1}}{n+1} - \frac{1}{n+1} \cdot \frac{x^{n+1}}{n+1} + C$$

$$= x^{n+1} \left(\frac{\ln(x)}{n+1} - \frac{1}{(n+1)^2} \right) + C, \quad C \in \mathbb{R}$$

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(3)

$$\int (x-2)^3 dx \qquad t = x-2,\ dt = dx$$

$$= \int t^3 dt$$

$$= \frac{t^4}{4}$$

$$= \frac{(x-2)^4}{4}$$

$u = \ln(x-2)$ | $dv = (x-2)^3$

$du = \frac{1}{x-2} dx$ | $v = \frac{(x-2)^4}{4}$

$$\int (x-2)^3 \cdot \ln(x-2) dx = \ln(x-2) \cdot \frac{(x-2)^4}{4} - \int \frac{(x-2)^4}{4} \cdot \frac{1}{(x-2)} dx$$

$$= \frac{\ln(x-2) \cdot (x-2)^4}{4} - \frac{1}{4} \int (x-2)^3 dx$$

$$= \ln(x-2)(x-2)^4/4 - \frac{1}{4} \cdot (x-2)^4/4 + C, \quad C \in \mathbb{R}$$

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4. $\int \frac{\ln(x^2)}{x^2} dx$

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7. $\int x^3 \ln(x^3) dx$

(4)

$u = \ln(x^2)$ $\quad dv = x^{-2} dx$

$du = \frac{1}{x^2} \cdot 2x = \frac{2}{x} dx$ $\quad v = \frac{x^{-1}}{-1} = -x^{-1}$

$$\int \frac{\ln(x^2)}{x^2} dx = \underbrace{\ln(x^2)}_{u} \cdot \underbrace{-x^{-1}}_{v} - \int \underbrace{-\frac{1}{x}}_{v} \cdot \underbrace{\frac{2}{x} dx}_{du}$$

$$= -\frac{\ln(x^2)}{x} + 2\int \frac{1}{x^2} dx$$

$$= -\frac{\ln(x^2)}{x} + 2\int x^{-2} dx$$

$$= -\frac{\ln(x^2)}{x} + 2 \cdot \frac{x^{-1}}{-1} + C$$

$$= -\frac{\ln(x^2)}{x} - \frac{2}{x} + C, \quad C \in \mathbb{R}$$

1. $\int x \ln(x) dx$

2. $\int x^n \ln(x) dx$

3. $\int (x-2)^3 \ln(x-2) dx$

4. $\int \frac{\ln(x^2)}{x^2} dx$

5. $\int \ln(2x) dx$

6. $\int x^3 \ln(3x) dx$

7. $\int x^3 \ln(x^3)\, dx$

(5)

$u = \ln(2x)$ | $dv = 1 \cdot dx$

$du = \frac{1}{2x} \cdot 2 = \frac{1}{x} dx$ | $v = x$

$$\int \ln(2x)\, dx = \underset{u}{\ln(2x)} \cdot \underset{v}{x} - \int \underset{v}{x} \cdot \underset{du}{\frac{1}{x} dx}$$

$$= \ln(2x) \cdot x - \int 1\, dx$$

$$= \ln(2x) \cdot x - x + C$$

$$= x(\ln(2x) - 1) + C, \quad C \in \mathbb{R}$$

1. $\int x \ln(x) dx$

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3. $\int (x-2)^3 \ln(x-2) dx$

4. $\int \frac{\ln(x^2)}{x^2} dx$

5. $\int \ln(2x) dx$

6. $\int x^3 \ln(3x) dx$

7. $\int x^3 \ln(x^3)\, dx$

(6)

$u = \ln(3x)$ | $dv = x^3\,dx$

$du = \frac{1}{3x} \cdot 3 = \frac{1}{x}\,dx$ | $v = \frac{x^4}{4}$

$$\int x^3 \cdot \ln(3x)\,dx = \ln(3x) \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x}\,dx$$

$$= \ln(3x) \cdot \frac{x^4}{4} - \frac{1}{4}\int x^3\,dx$$

$$= \ln(3x) \cdot \frac{x^4}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + C$$

$$= x^4\left(\frac{\ln(3x)}{4} - \frac{1}{16}\right) + C, \quad C \in \mathbb{R}$$

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7. $\int x^3 \ln(x^3)\, dx$

(7)

$u = \ln(x^3)$ | $dv = x^3 dx$

$du = \frac{1}{x^3} \cdot 3x^2 = \frac{3}{x} dx$ | $v = \frac{x^4}{4}$

$$\int x^3 \ln(x^3) dx = \underbrace{\ln(x^3)}_{u} \cdot \underbrace{\frac{x^4}{4}}_{v} - \int \underbrace{\frac{x^4}{4}}_{v} \cdot \underbrace{\frac{3}{x} dx}_{du}$$

$$= \ln(x^3) \cdot \frac{x^4}{4} - \frac{3}{4} \int x^3 dx$$

$$= \ln(x^3) \cdot \frac{x^4}{4} - \frac{3}{4} \cdot \frac{x^4}{4} + C \quad \text{CEIR}$$

$$= x^4 \left(\frac{\ln(x^3)}{4} - \frac{3}{16} \right) + C$$

$$= \frac{x^4}{16} \left(4 \ln(x^3) - 3 \right) + C \quad , \text{CEIR}$$

8. $\int \frac{\ln(\sqrt{x})}{\sqrt{x}} dx = \int \ln(x^{1/2}) \cdot x^{-1/2} dx$

9. $\int x \log(x) dx$

10. $\int xe^x dx$

11. $\int xe^{mx} dx$

12. $\int (2x+1)e^{3x} dx$

13. $\int \ln(x^x)\, dx$

14. $\int x^2 e^x dx$

15. $\int x^3 e^{x^2} dx$

$u = \ln(x^{1/2})$

$du = \frac{1}{x^{1/2}} \cdot \frac{1}{2} x^{-1/2} dx$

$= \frac{1}{2} x^{-1} dx$

$dv = x^{-1/2} dx$

$v = \frac{x^{1/2}}{1/2} = 2x^{1/2}$

$\ln(x^{1/2}) \cdot 2x^{1/2} - \int 2x^{1/2} \cdot \frac{1}{2} x^{-1} dx$

$= 2\ln(x^{1/2}) \cdot x^{1/2} - \int x^{-1/2} dx$

$= 2\ln(x^{1/2}) \cdot x^{1/2} - 2x^{1/2} + C, \quad C \in \mathbb{R}$

8. $\int \frac{\ln(\sqrt{x})}{\sqrt{x}} dx$

9. $\int x \log(x) dx$

10. $\int xe^x ax$

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