Ejemplos resueltos Unidad 4

1. Sean $Z_1 = 2 - 3i$, $Z_2 = 4 - 5i$

Determine: $Z_1 \bullet Z_2$

Solución:

$Z_1 \bullet Z_2 = (2 - 3i) \bullet (4 - 5i)$

$Z_1 \bullet Z_2 = -7 - 22i$

2. Sean $Z_1 = 2 + 5i$, $Z_2 = 1 - 3i$

Determine: $\frac{Z_1}{Z_2}$

Solución:

$$\frac{Z_1}{Z_2} = \frac{2 + 5i}{1 - 3i}$$

$$= \frac{2 + 5i}{1 - 3i} \bullet \frac{1 + 3i}{1 + 3i}$$

$$= \frac{(2 + 5i) \bullet (1 + 3i)}{(1 - 3i) \bullet (1 + 3i)}$$

$$= \frac{-13 + 11i}{10}$$

$$= \frac{-13}{10} + \frac{11i}{10}$$

3. Sean $Z_1 = 1 + i$, $Z_2 = 3 + i$

Determine la parte real de $W = i\dfrac{Z_1}{Z_2} - 1$

Solución:

$$\frac{Z_1}{Z_2} = \frac{1+i}{3+i}$$

$$= \frac{1+i}{3+i} \bullet \frac{3-i}{3-i}$$

$$= \frac{(1+i)\bullet(3-i)}{(3+i)\bullet(3-i)}$$

$$= \frac{4+2i}{10}$$

$$= \frac{4}{10} + \frac{2i}{10}$$

$$= \frac{2}{5} + \frac{i}{5}$$

$$i\frac{Z_1}{Z_2} = i\left(\frac{2}{5} + \frac{i}{5}\right) = -\frac{1}{5} + \frac{2}{5}i$$

$$W = i\frac{Z_1}{Z_2} - 1 = -\frac{1}{5} + \frac{2}{5}i - 1$$

$$= -\frac{6}{5} + \frac{2}{5}i$$

$$\operatorname{Re}(W) = -\frac{6}{5}$$

4. Sean $Z_1 = 1 - 6i \quad , \quad Z_2 = 2 - 5i$

Determine la parte imaginaria de $W = \dfrac{\overline{Z_1}}{Z_2} - 2i$

Solución:

$$\overline{Z_1} = 1 + 6i$$

$$\frac{\overline{Z_1}}{Z_2} = \frac{1+6i}{2-5i}$$

$$= \frac{1+6i}{2-5i} \bullet \frac{2+5i}{2+5i}$$

$$= \frac{(1+6i)\bullet(2+5i)}{(2-5i)\bullet(2+5i)}$$

$$= \frac{-28+17i}{29}$$

$$= \frac{-28}{29} + \frac{17i}{29}$$

$$W = \frac{\overline{Z_1}}{Z_2} - 2i$$

$$W = \frac{-28}{29} + \frac{17i}{29} - 2i$$

$$W = -\frac{28}{29} - \frac{41}{29}i$$

$$\mathrm{Im}(W) = -\frac{41}{29}$$

5. Sean $Z_1 = 2i$, $Z_2 = 1 - i$

Determine el módulo de $W = \dfrac{\overline{Z_1}}{Z_2} - \dfrac{\overline{Z_2}}{Z_1}$

Solución:

$\overline{Z_1} = -2i$

$$\frac{\overline{Z_1}}{Z_2} = \frac{-2i}{1-i}$$

$$= \frac{-2i}{1-i} \bullet \frac{1+i}{1+i}$$

$$= \frac{(-2i) \bullet (1+i)}{(1-i) \bullet (1+i)}$$

$$= \frac{2-2i}{2} = 1 - i$$

$\overline{Z_2} = 1 + i$

$$\frac{\overline{Z_2}}{Z_1} = \frac{1+i}{2i}$$

$$= \frac{1+i}{2i} \bullet \frac{-2i}{-2i}$$

$$= \frac{(1+i) \bullet (-2i)}{2i \bullet (-2i)}$$

$$= \frac{2-2i}{4}$$

$$= \frac{1}{2} - \frac{1}{2}i$$

$$W = \frac{\overline{Z_1}}{Z_2} - \frac{\overline{Z_2}}{Z_1}$$

$$W = 1 - i - \left(\frac{1}{2} - \frac{1}{2}i\right)$$

$$W = \frac{1}{2} - \frac{1}{2}i$$

$$\|W\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \frac{\sqrt{2}}{2}$$

6. Resuelva en C la siguiente ecuación:

$$x^2 - 8x + 41 = 0$$

Solución:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{8 \pm \sqrt{64 - 164}}{2}$$

$$x = \frac{8 \pm \sqrt{-100}}{2}$$

$$x = \frac{8 \pm 10i}{2}$$

$$x_1 = \frac{8 + 10i}{2} = 4 + 5i$$

$$x_2 = \frac{8 - 10i}{2} = 4 - 5i$$

7. Resuelva en C la siguiente ecuación:

$$x^2 - 6x + 58 = 0$$

Solución:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{36 - 232}}{2}$$

$$x = \frac{6 \pm \sqrt{-196}}{2}$$

$$x = \frac{6 \pm 14i}{2}$$

$$x_1 = \frac{6 + 14i}{2} = 3 + 7i$$

$$x_2 = \frac{6 - 14i}{2} = 3 - 7i$$

8. Exprese en la forma binomial: $Z = 2cis\left(\frac{\pi}{5}\right) \bullet 3cis\left(\frac{\pi}{20}\right)$

Solución:

$$Z = 6cis\left(\frac{\pi}{5} + \frac{\pi}{20}\right)$$

$$Z = 6cis\left(\frac{\pi}{4}\right)$$

$$Z = 6\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$$

$$Z = 6\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$Z = 3\sqrt{2} + 3i\sqrt{2}$$

9. Exprese en la forma binomial: $Z = \dfrac{cis\left(\frac{10\pi}{21}\right)}{cis\left(\frac{\pi}{7}\right)} + cis(\pi)$

Solución:

$$Z = \frac{cis\left(\frac{10\pi}{21}\right)}{cis\left(\frac{\pi}{7}\right)} + cis(\pi)$$

$$Z = cis\left(\frac{10\pi}{21} - \frac{\pi}{7}\right) + \cos(\pi) + i\sin(\pi)$$

$$Z = cis\left(\frac{\pi}{3}\right) - 1$$

$$Z = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right) - 1$$

$$Z = \frac{1}{2} + i\frac{\sqrt{3}}{2} - 1$$

$$Z = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

10. Exprese en la forma polar el complejo $\overline{Z}$ si

$$Z = \frac{2}{\sqrt{3}+i}$$

Solución:

$$Z = \frac{2}{\sqrt{3}+i}$$

$$Z = \frac{2}{\sqrt{3}+i} \bullet \frac{\sqrt{3}-i}{\sqrt{3}-i}$$

$$Z = \frac{2\left(\sqrt{3}-i\right)}{\left(\sqrt{3}+i\right)\left(\sqrt{3}-i\right)}$$

$$Z = \frac{2\left(\sqrt{3}-i\right)}{4}$$

$$Z = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

$$\overline{Z} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

$$\|\overline{Z}\| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1$$

$$\left.\begin{array}{r} \|\overline{Z}\|\cos(\theta) = \dfrac{\sqrt{3}}{2} \\ \|\overline{Z}\|\sin(\theta) = \dfrac{1}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{r} \cos(\theta) = \dfrac{\sqrt{3}}{2} \\ \sin(\theta) = \dfrac{1}{2} \end{array}\right\} \Rightarrow \theta = \frac{\pi}{6}$$

pues $\overline{Z}$ está ubicado en el primer cuadrante

$$\overline{Z} = \|\overline{Z}\| cis\left(\frac{\pi}{6}\right) = cis\left(\frac{\pi}{6}\right)$$