Prueba Matemática
Curso: 2do ECO
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1)

$y = \operatorname{sen}(\ln x) + \cos(\ln x)$

$y'' \cdot x^2 + y' \cdot x + y = 0$

$y' = \cos(\ln(x)) \cdot \frac{1}{x} - \operatorname{sen}(\ln(x)) \cdot \frac{1}{x}$

$y' = \left(\frac{\cos(\ln(x)) - \operatorname{sen}(\ln(x))}{x}\right)$

$y'' = \frac{\left[\left(-\operatorname{sen}(\ln(x)) \cdot \frac{1}{\cancel{x}} - \cos(\ln(x)) \cdot \frac{1}{\cancel{x}}\right) \cdot \cancel{x}\right] - \left[\cos(\ln(x)) - \operatorname{sen}(\ln(x)) \cdot 1\right]}{x^2}$

$y'' = \frac{\cancel{-\operatorname{sen}(\ln(x))} - \cos(\ln(x)) - \cos(\ln(x)) \cancel{+ \operatorname{sen}(\ln(x))}}{x^2}$

$y'' = \frac{-2\cos(\ln(x))}{x^2}$

$\therefore \frac{-2\cos(\ln(x))}{\cancel{x^2}} \cdot \cancel{x^2} + \left(\frac{\cos(\ln(x)) - \operatorname{sen}(\ln(x))}{\cancel{x}}\right) \cdot \cancel{x} = 0$

$-2\cos(\ln(x)) + \cos(\ln(x)) - \operatorname{sen}(\ln(x)) + y = 0$

$\cancel{-\cos(\ln(x))} - \cancel{\operatorname{sen}(\ln(x))} + \cancel{\operatorname{sen}(\ln(x))} + \cancel{\cos(\ln(x))} = 0$

$0 = 0$

2) a) $y = 3^{\tan(x^3+x)}$ / ln

$\ln(y) = \tan(x^3+x) \cdot \ln(3)$ / $\frac{d_y}{d_x}$

$\frac{1}{y} \cdot y' = \left[\left(\sec^2(x^3+x)\cdot(3x^2+1)\right)\cdot \ln(3)\right] + \cancel{\left[\tan(x^3+x)\cdot \frac{1}{3}\cdot 0\right]}$

$\frac{1}{y} \cdot y' = \left[\sec^2(x^3+x)\cdot(3x^2+1)\cdot \ln(3)\right]$

$y' = y\left[\sec^2(x^3+x)\cdot(3x^2+1)\cdot \ln(3)\right]$

$y' = 3^{\tan(x^3+x)}\left[\sec^2(x^3+x)\cdot(3x^2+1)\cdot \ln(3)\right]$

b) $y = \frac{(e^x - x)^{10}\cdot(8+x^3)^2}{\sqrt{6x-\sqrt{x}}}$ /· ln

$\ln(y) = 10\ln(e^x - x) + 2\ln(8+x^3) - \ln\left(\sqrt{6x-\sqrt{x}}\right)$ / $\frac{d_y}{d_x}$

$\frac{1}{y}\cdot y' = 10\cdot\frac{1}{e^x-x}\cdot(e^x-1) + 2\cdot\frac{1}{8+x^3}\cdot(3x^2) - \frac{1}{\sqrt{6x-\sqrt{x}}}\cdot\frac{1}{2\sqrt{6x-\sqrt{x}}}$

$\frac{1}{y}\cdot y' = \frac{10(e^x-1)}{e^x-x} + \frac{2(3x^2)}{8+x^3} - \left(\sqrt{6x-\sqrt{x}}\cdot 2\sqrt{6x-\sqrt{x}}\right)^{-1}$

$y' = y\left[\frac{10(e^x-1)}{e^x-x} + \frac{2(3x^2)}{8+x^3} - \left(\sqrt{6x-\sqrt{x}}\cdot 2\sqrt{6x-\sqrt{x}}\right)^{-1}\right]$

$y' = \left(\frac{(e^x-x)^{10}\cdot(8+x^3)^2}{\sqrt{6x-\sqrt{x}}}\right)\cdot\left[\frac{10(e^x-1)}{e^x-x} + \frac{2(3x^2)}{8+x^3} - \left(\sqrt{6x-\sqrt{x}}\cdot 2\sqrt{6x-\sqrt{x}}\right)^{-1}\right]$

2.C

$$y = \frac{sen(x) \cdot (x^2+x)^{3x}}{110} \quad / \ln$$

$$\ln(y) = \ln(sen(x)) + 3x\ln(x^2+x) - \ln(110) \quad / \frac{d_y}{dx}$$

$$\frac{1}{y} \cdot y' = \left(\frac{1}{sen(x)} \cdot \cos(x)\right) + \left(3x\ln(x) + 6x\ln(x)\right) - \left(\frac{1}{\cancel{110}} \cdot \cancel{0}\right)$$

$$\frac{1}{y} \cdot y' = \cot g(x) + \cancel{3x} \cdot \frac{1}{\cancel{x}} + 6\cancel{x} \cdot \frac{1}{\cancel{x}}$$

$$y' = y\left(\cot g(x) + 9\right)$$

$$y' = \left(\frac{sen(x) \cdot (x^2+x)^{3x}}{110}\right)\left(\cot g + 9\right)$$

③ a) $y^2 = x^3 + sen(xy) - x\ln(y)$

$$2y \cdot y' = 3x^2 + \cos(xy)\cdot(1\cdot y') - \left[\left(1\cdot \ln(y)\right) + \left(x \cdot \frac{1}{y}\cdot y'\right)\right]$$

$$2y\cdot y' = 3x^2 + y'\cos(xy) - \ln(y) - \frac{xy'}{y}$$

$$2y\cdot y' - y'\cos(xy) + \frac{xy'}{y} = 3x^2 - \ln(y)$$

$$y'\left(2y - \cos(xy) + \frac{x}{y}\right) = 3x^2 - \ln(y)$$

$$y' = \frac{3x^2 - \ln(y)}{2y - \cos(xy) + \frac{x}{y}}$$

b) $y + x\cdot\sqrt{1+2y} = x^2$ Determine $y'(1,0)$

$$y' + \left((1\sqrt{1+2y}) + x\cdot\frac{1}{2\sqrt{1+2y}}\cdot 2y'\right) = 2x$$

$$y' + \sqrt{1+2y} + \frac{\cancel{2}y'x}{\cancel{2}\sqrt{1+2y}} = 2x$$

$$y' + \frac{\sqrt{1+2y}^{\,2} + y'x}{\sqrt{1+2y}} = 2x$$

$$y' + \frac{1+2y+y'x}{\sqrt{1+2y}} = 2x \quad / \cdot\sqrt{1+2y}$$

$\hookrightarrow y' \cdot \sqrt{1+2y} + 1 + 2y + y'x = 2x \cdot \sqrt{1+2y}$

$y' \cdot \sqrt{1+2y} + y'x = 2x \cdot \sqrt{1+2y} - 2y - 1$

$y' \left(\sqrt{1+2y} + x \right) = 2x \cdot \sqrt{1+2y} - 2y - 1$

$$y' = \frac{2x \cdot \sqrt{1+2y} - 2y - 1}{\sqrt{1+2y} + x}$$

$$\therefore y' = \frac{2 \cdot \sqrt{1} - 1}{\sqrt{1+0} + 1} \Bigg\} \frac{2-1}{1+1} = \frac{1}{2}$$

R: $y' = \frac{1}{2}$ en el punto $(1, 0)$

Pregunta 4

a) $3x^2y + y^2 - 3x = 1$ $P(1,-4)$

$(6x\cdot y)+(3x^2\cdot y')+2y\cdot y'-3=1$

$6xy + 3x^2y' + 2y\cdot y' - 3 = 1$

$y'(3x^2+2y) = 3-6xy$

$y' = \dfrac{3-6xy}{3x^2+2y}$ $/P(1,-4)$

$y' = \dfrac{3-6\cdot 1\cdot -4}{3\cdot 1^2+2\cdot -4}$ $y' = \dfrac{3+24}{3-8} = \dfrac{-27}{5}$

$(y+4) = \dfrac{-27}{5}(x-1)$

$y = \dfrac{-27}{5} + \dfrac{27}{5} - 4$

$$\boxed{y = \frac{-27x}{5} + \frac{7}{5}}$$

(5) $y = x^3 - 8x + 2$ $\therefore \frac{dy}{dx}$ $m_t = 4$

$y' = 3x^2 - 8 + 0$

$y' = 3x^2 - 8$

$4 = 3x^2 - 8$

$12 = 3x^2$

$\sqrt{4} = x$ } $x = 2$

$\therefore y = 8 - 16 + 2$

$y = -6$

$m_t = 4 \Rightarrow m_n = \frac{-1}{4}$

$P(2, -6)$

$\therefore (y + 6) = \frac{-1}{4}(x - 2)$

$y = \frac{-1x}{4} + \frac{2}{4} - 6$ } $y = \frac{-1x}{4} + \frac{1}{2} - 6$

$$\boxed{y = \frac{-x}{4} - \frac{11}{2}}$$