Sergio Yansen Núñez

Calcule, si existen, los siguientes límites:

1. $\lim\limits_{x\to 1^+} \frac{xln(x)}{x+xln(x)-1}$

2. $\lim\limits_{x\to \left(\frac{\pi}{2}\right)^+} \frac{ln(sen(x))}{cot(x)}$

3. $\lim\limits_{x\to 1} \frac{cos(ln(x)) - x^2}{e^{2x-1} - e}$

4. $\lim\limits_{x\to 0} \frac{ln(x^2+1)}{e^{2x}-1}$

5. $\lim\limits_{x\to \pi} \frac{sen(\pi cos(x))+x-\pi}{x^2-\pi^2}$

6. $\lim\limits_{x\to 0} \frac{1+cos(\pi cos(x^2))}{ln(x^2+1)}$

7. $\lim\limits_{x\to 0} \frac{x-arctan(x)}{x-sen(x)}$

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1. $\lim\limits_{x\to 1^+} \frac{xln(x)}{x+xln(x)-1}$ forma indeterminada del tipo $0/0$

$= \lim\limits_{x\to 1^+} \frac{1+ln(x)}{2+ln(x)} = \frac{1}{2}$

2. $\lim\limits_{x\to\left(\frac{\pi}{2}\right)^+} \frac{ln(sen(x))}{cot(x)}$ forma indeterminada del tipo $0/0$

$= \lim\limits_{x\to\left(\frac{\pi}{2}\right)^+} \frac{\frac{1}{sen(x)}\cdot cos(x)}{-cosec^2(x)} = \lim\limits_{x\to\left(\frac{\pi}{2}\right)^+} (-cos(x)\cdot sen(x)) = 0$

3. $\lim\limits_{x\to 1} \frac{cos(ln(x))-x^2}{e^{2x-1}-e}$ Forma indeterminada $\frac{0}{0}$, aplicando L' H

$= \lim\limits_{x\to 1} \frac{-sen(ln(x))\cdot\frac{1}{x}-2x}{e^{2x-1}\cdot 2}$

$= \frac{-2}{e\cdot 2}$

$= -\frac{1}{e}$

4. $\lim\limits_{x\to 0} \frac{ln(x^2+1)}{e^{2x}-1}$ Forma indeterminada $\frac{0}{0}$. Por $L'H\hat{o}pital$

$= \lim\limits_{x\to 0} \frac{\frac{1}{x^2+1}\cdot 2x}{e^{2x}\cdot 2}$

$= \lim\limits_{x\to 0} \frac{x}{e^{2x}(x^2+1)} = 0$

5. $\lim\limits_{x\to\pi} \frac{sen(\pi cos(x))+x-\pi}{x^2-\pi^2}$ Forma indeterminada $\frac{0}{0}$. Por $L'H\hat{o}pital$

$= \lim\limits_{x\to\pi} \frac{cos(\pi cos(x))\cdot(-\pi sen(x))+1}{2x} = \frac{1}{2\pi}$

6. $\lim\limits_{x\to 0}\frac{1+cos(\pi cos(x^2))}{ln(x^2+1)}$ (Forma indeterminada del tipo $\frac{0}{0}$), por $L'H\widehat{o}pital$ se obtiene:

$$\lim_{x\to 0}\frac{1+cos(\pi cos(x^2))}{ln(x^2+1)} = \lim_{x\to 0}\frac{-sen(\pi cos(x^2))\cdot\pi(-sen(x^2)\cdot 2x}{\frac{1}{x^2+1}\cdot 2x}$$

$$= \lim_{x\to 0}\frac{\pi sen(\pi cos(x^2))sen(x^2)}{\frac{1}{x^2+1}} = 0$$

7. $\lim\limits_{x\to 0}\frac{x-arctan(x)}{x-sen(x)}$ (Forma indeterminada del tipo $\frac{0}{0}$), por $L'H\widehat{o}pital$ se obtiene:

$$\lim_{x\to 0}\frac{x-arctan(x)}{x-sen(x)} = \lim_{x\to 0}\frac{1-\frac{1}{1+x^2}}{1-cos(x)}$$

$$= \lim_{x\to 0}\frac{\frac{1+x^2-1}{1+x^2}}{1-cos(x)}$$

$$= \lim_{x\to 0}\frac{\frac{x^2}{1+x^2}}{1-cos(x)}$$

$$= \lim_{x\to 0}\frac{x^2}{(1+x^2)(1-cos(x))}$$

$$= \lim_{x\to 0}\left(\frac{x^2}{1-cos(x)}\cdot\frac{1}{1+x^2}\right)$$

$$\lim_{x\to 0}\frac{1}{1+x^2} = 1$$

$\lim\limits_{x\to 0}\frac{x^2}{1-cos(x)}$ (Forma indeterminada del tipo $\frac{0}{0}$), por $L'H\widehat{o}pital$ se obtiene:

$$\lim_{x\to 0}\frac{x^2}{1-cos(x)} = \lim_{x\to 0}\frac{2x}{sen(x)} = \lim_{x\to 0}\frac{2}{\frac{sen(x)}{x}} = 2$$

Luego, $\lim\limits_{x\to 0}\left(\frac{x^2}{1-cos(x)}\cdot\frac{1}{1+x^2}\right) = 2\cdot 1 = 2$